Nuclear Physics 11

The linear-σ model,
the Goldstone theorem
&
chiral symmetry

RECAP:

→ Naturalness IK

fine-tuning

indicates a conspiracy (that is, a symmetry)

$\frac{m_\rho}{m_N} \sim \frac{4}{5}$ $\frac{m_\pi}{m_N} \sim \frac{1}{7}$

→ Separation of Scales can be exploited in our favor

RGEs / EFTs / ....

→ All converges towards a possible EFT for hadrons

$\boxed{\text{SPOILER}}$ $\Rightarrow$ this EFT is called Chiral Perturbation Theory (ChPT)

But before this let's go back to the 60's ...

... and steal Gell-Mann & Levi's manuscript

Gell-Mann & Levi → $\boxed{\text{Why pion so light?}}$

a formidable problem

→ Let's try to give a partner to the pion!

1) Theory w/ nucleons, a scalar & 3 pseudoscalars

$$N, \phi^0, \vec{\phi} = \{\phi_1, \phi_2, \phi_3\}$$

$$\mathcal{L} = i\bar{N}\not{\partial}N + g\bar{N}(\phi_0 + i\gamma^5\vec{\tau}\cdot\vec{\phi})N$$

$$+ \frac{1}{2}\sum_i \partial_\mu\phi_i \partial^\mu\phi_i - V(\phi)$$

$$V(\phi) = \frac{1}{2}\mu^2\sum_i \phi_i^2 + \frac{\lambda}{4}\Big(\sum_i \phi_i^2\Big)^2$$

Important detail:

$$\boxed{N \text{ massless}, \; \Phi^0, \overline{\Phi}^0 \text{ mass } \mu}$$

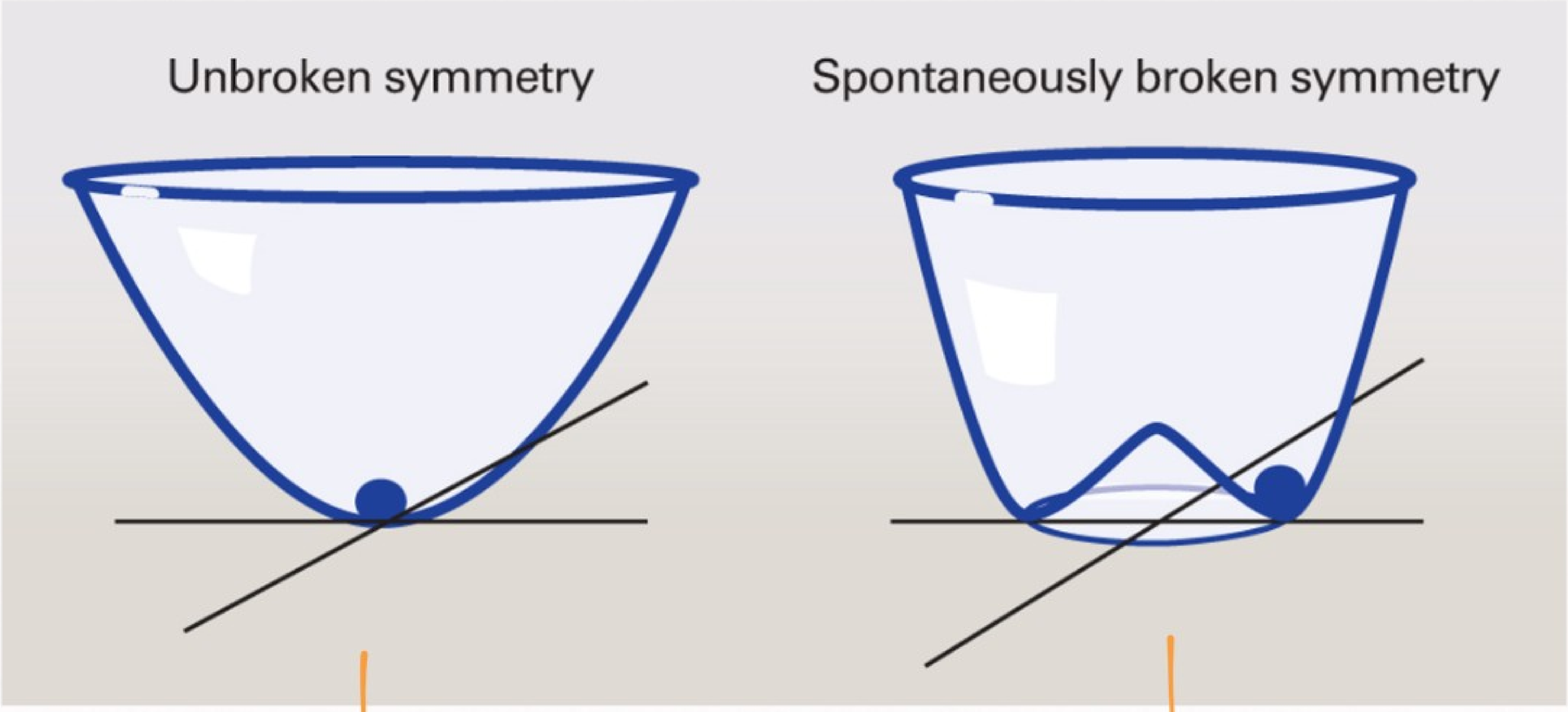
$$\tfrac{1}{2}^- \qquad\qquad 0^+ \quad 0^- \leftarrow J^P$$

Except if magic happens...

But we need magic w/ a mexican touch!

Have you heard about the mexican hat potential?

$$V(\phi) = \frac{1}{2}\mu^2\left(\sum_i \phi_i^2\right) + \frac{\lambda}{4}\left(\sum_i \phi_i^2\right)^2$$



(a) $\boxed{\mu^2 > 0, \lambda > 0}$

(b) $\boxed{\mu^2 < 0, \lambda > 0}$

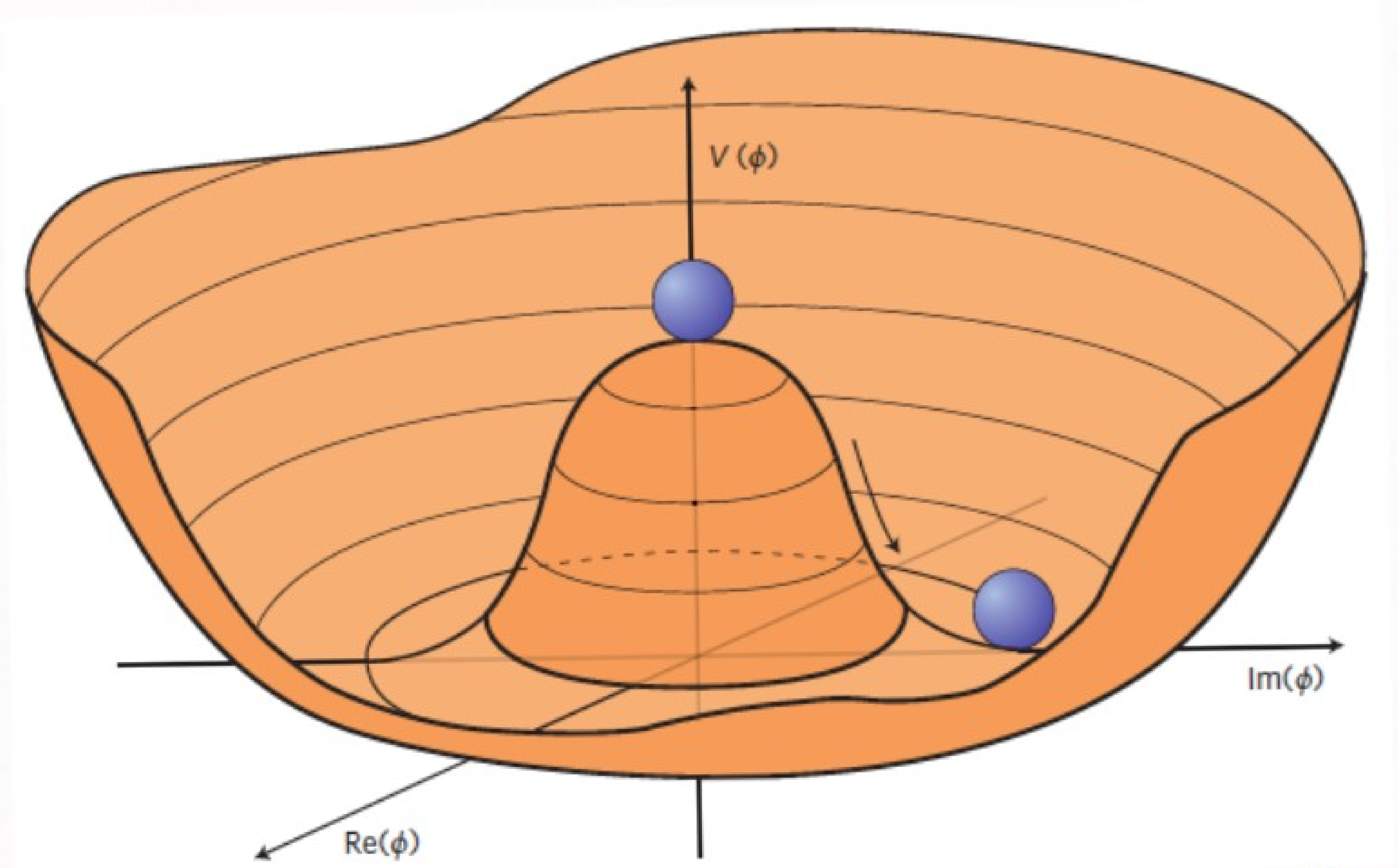
a) Standard Quartic potential

$\mu^2 > 0, \lambda > 0$

$\left[\Rightarrow N \text{ massless} \atop \phi_0, \vec{\phi} \text{ mass } \mu \right]$

b) Mexican hat potential

$\mu^2 < 0, \lambda > 0 \Rightarrow [?]$



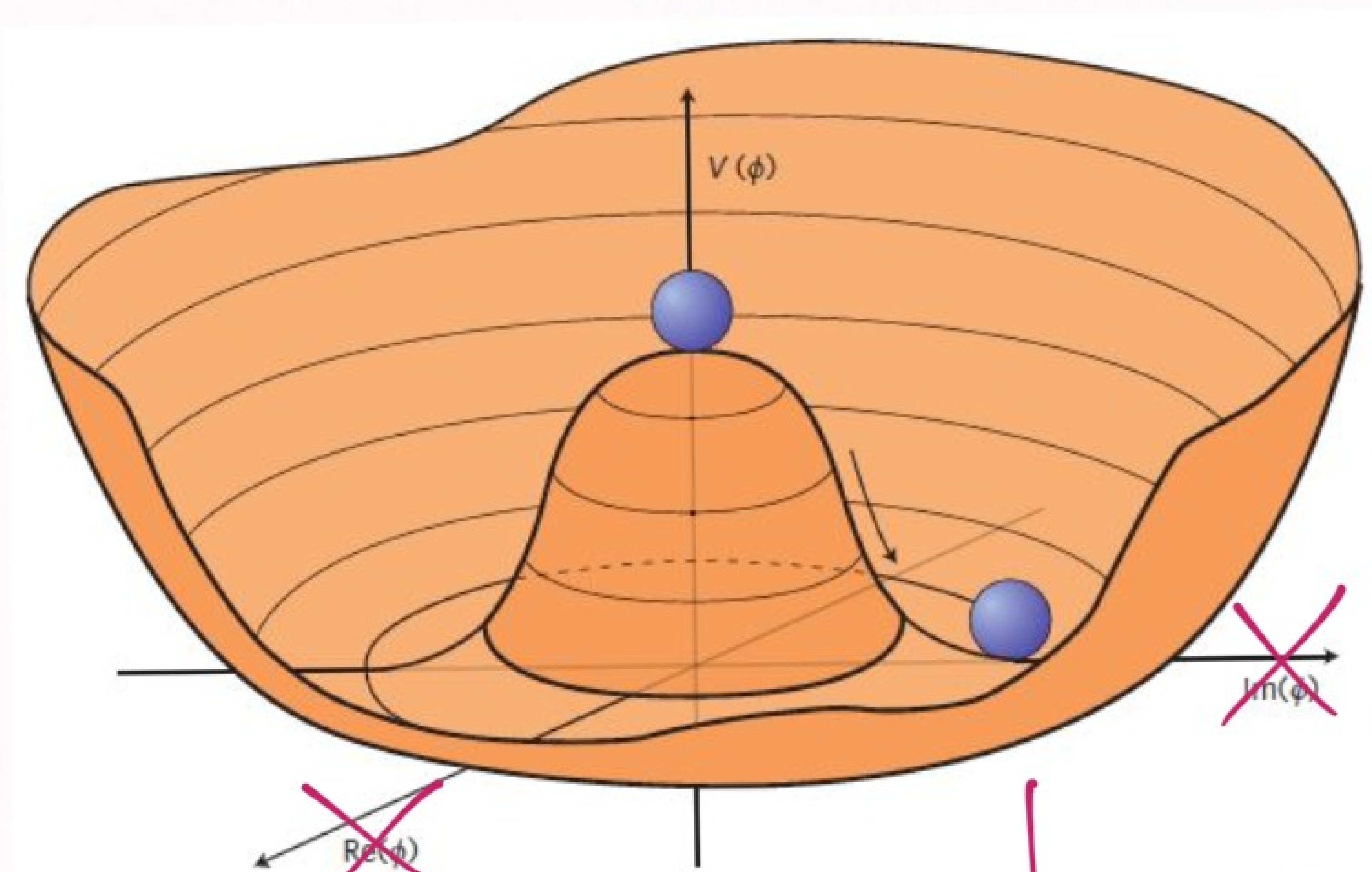
$\Rightarrow \mu^2 < 0$ can't be the mass of any field

$\Rightarrow$ What is going on here?

WHAT'S GOING ON HERE?

1) Fundamental state

$\simeq$ minimal energy state



ϕ^0

$|\vec{\phi}|$

$$\boxed{\frac{dV}{d\phi^0} = 0}$$

$$\phi^0 = v = \sqrt{-\frac{\mu^2}{\lambda}}$$

2) We only know to solve QFTs if we can write them as harmonic oscillators (+perturbations)

1) + 2) Let's expand this QFT around the minimum!

a) Old variables:

$\phi^0, \vec{\phi}$

b) New variables

$\sigma = \phi^0 - v \quad , \quad \vec{\pi} = \vec{\phi}$

$\Downarrow$

$$\mathcal{L} = \bar{N}(i\not\partial - \underbrace{gv}_{\boxed{1}})N + g\bar{N}(\sigma + i\gamma^5 \underbrace{\vec{\tau}\cdot\vec{\pi}}_{\boxed{2}})N$$

$$+ \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \underbrace{\frac{1}{2}(\lambda v^2)\sigma^2}_{\boxed{3}}$$

$$\underbrace{+ \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi}}_{\boxed{4}} - V(\vec{\pi}, \sigma)$$

(Equivalent to old Lagrangian)

NOTES:

$\boxed{1}$ → the nucleon has a mass now: $\boxed{M_N = gv}$

$\boxed{2}$ → the couplings of N to the sigma & pion are identical:

$$\boxed{g_{\pi NN} = g_{\sigma NN} = g}$$

($g_{\pi NN} \simeq 1.29\, g_{\sigma NN}$ in the real world)

$\boxed{3}$ → the sigma has a mass

$$\boxed{m_\sigma^2 = \lambda v^2 = -\mu^2}$$

$\boxed{4}$ → the pion is massless

1) Strong point:

explains why the pion is light

If we add $\Delta V(\phi) = -\frac{\epsilon}{v}\phi_0$

we can obtain a small pion mass

2) Weak point:

predicts a scalar meson

(the sigma)

→ not found experimentally

for many years

→ Gell-Mann & Levy proposed

the non-linear sigma model

(without sigmas)

→ But at the end we found

the sigma (plot twist)

Linear Sigma Model

Quintessential example of spontaneous symmetry breaking

Introduces the idea of the mexican hat potential

$\boxed{\text{GOLDSTONE THEOREM}}$

$\Downarrow$

Full generalization of the ideas contained in the linear sigma model

GOLDSTONE THEOREM

(schematic version)

1) Hamiltonian H invariant under group G

$[H, G] = 0$

2) $H \rightarrow |0\rangle$ vacuum state

3) But $|0\rangle$ not invariant under G

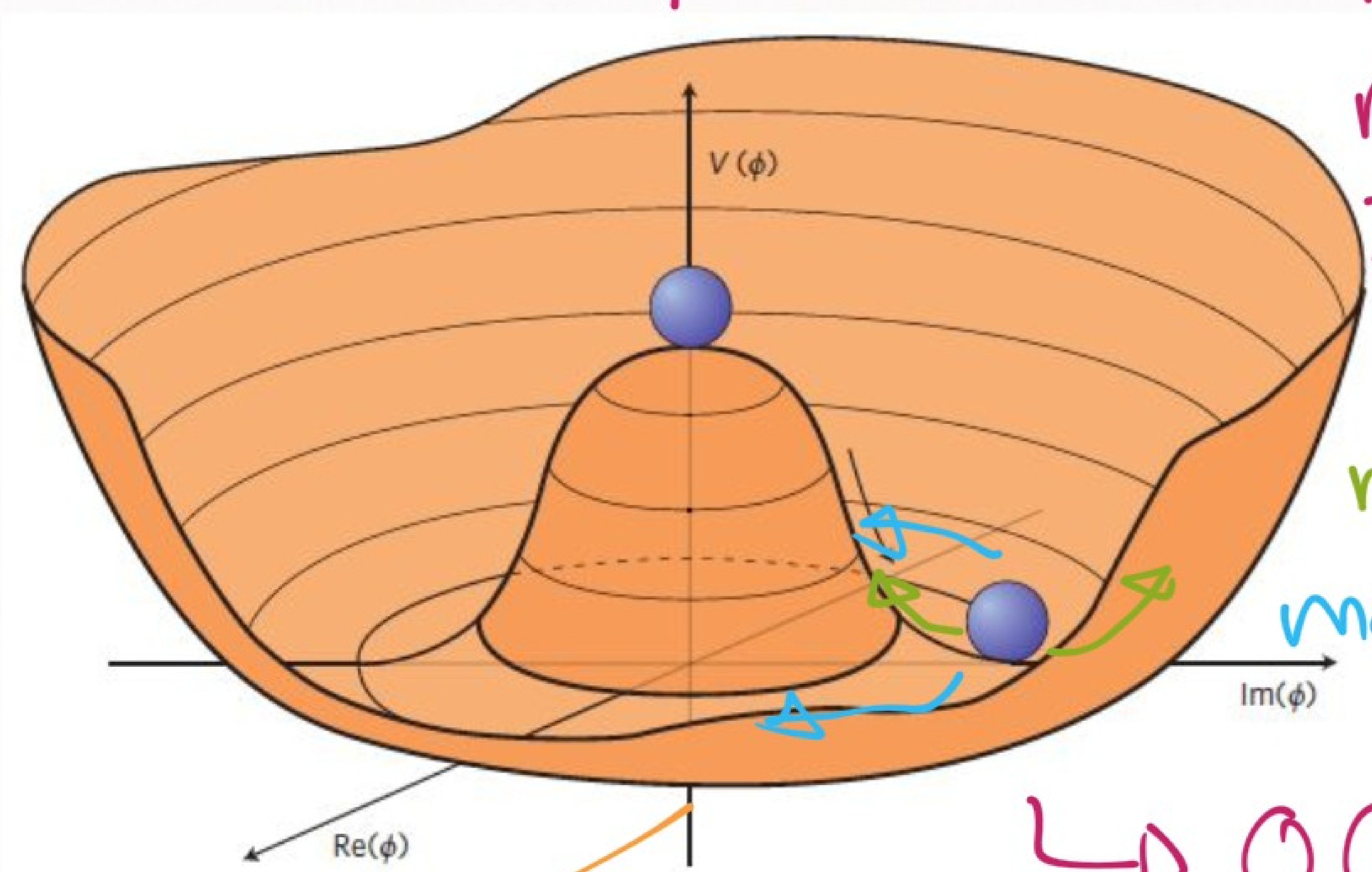
$\rightarrow$ invariant under $H \subset G$ (subgroup)

$\Rightarrow$ the quotient group $\frac{G}{H}$ can be use to generate states that are not the vacuum $|0\rangle$ but have the same energy

Example 1

→ $O(2)$

Fields:
$Re(\phi)$
$Im(\phi)$



massive

massless

→ $O(1)$

(trivial group)

We can move on the sides:

$$\frac{O(2)}{O(1)} \sim U(1) \quad (\text{or } O(2))$$

Originally → two massive fields

⇓

Result → a massive field

→ a massless Goldstone boson

Example 2 :

The linear sigma model

$$\left.\begin{array}{l} G = O(4) \\ H = O(3) \end{array}\right\} \to O(n) = \{ RR^T = 1 \,/\, |\det(R)| = 1 \}$$

$\frac{n(n-1)}{2}$ generators

$$\frac{G}{H} \to 6 - 3 = 3 \text{ generators}$$

$\Downarrow$

3 Goldstone bosons

$\Downarrow$

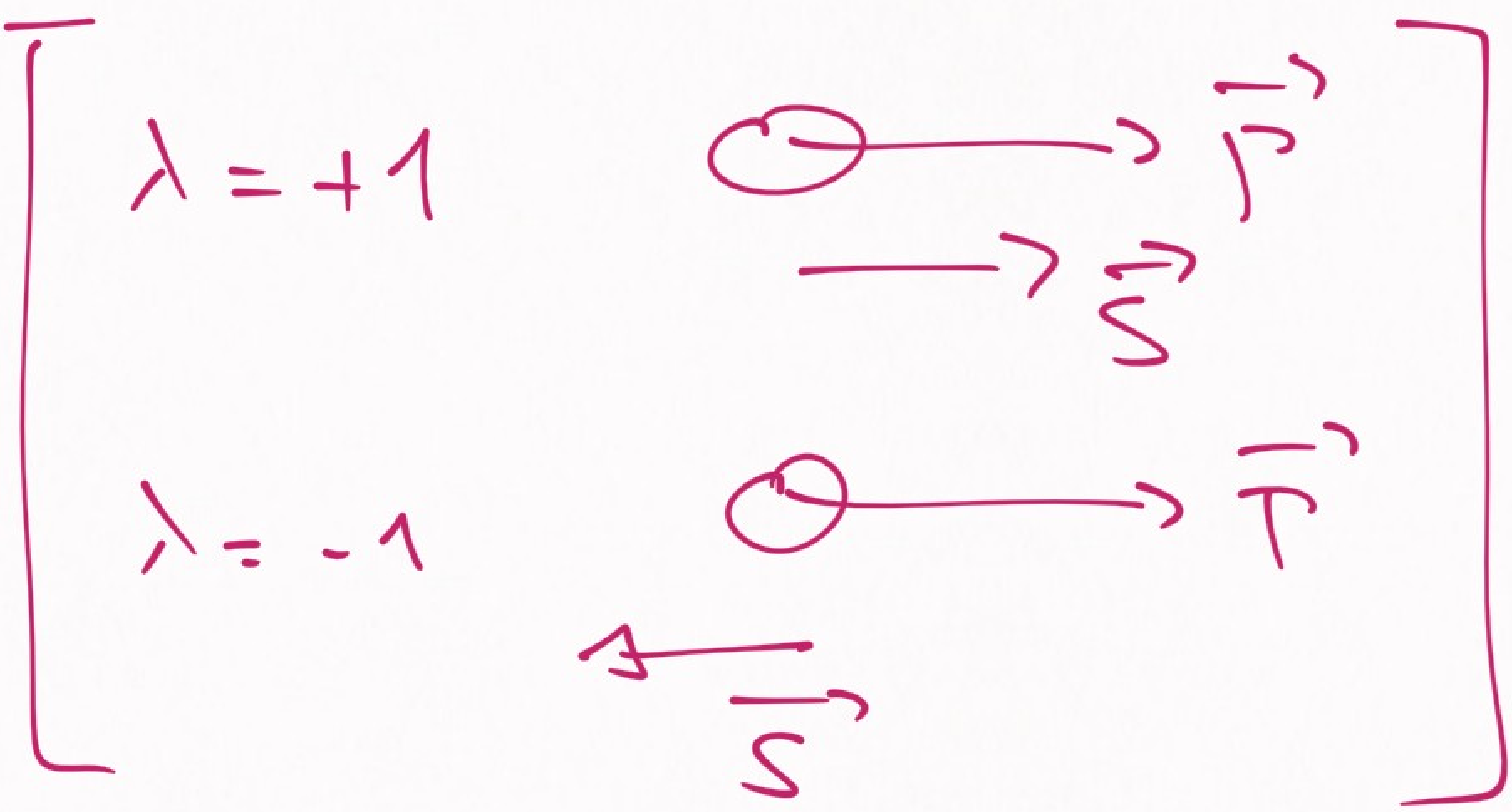
$\vec{\pi} = (\pi_1, \pi_2, \pi_3)$

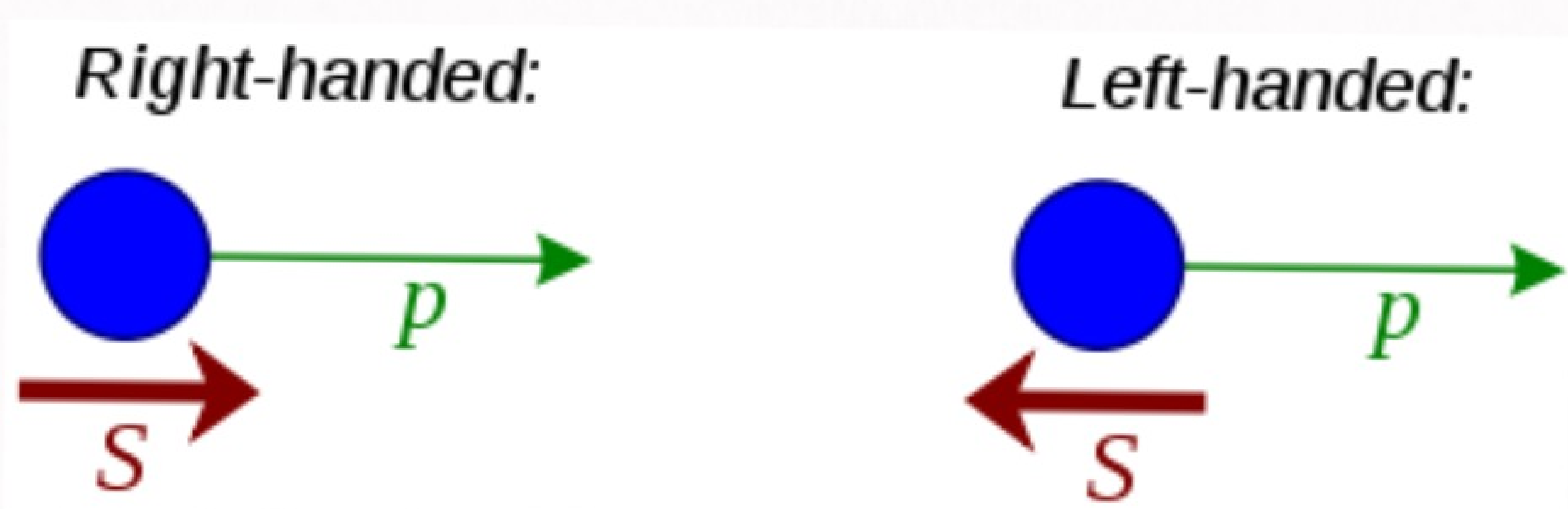
(3 pions)

$$\boxed{\text{CHIRAL SYMMETRY}}$$

1) Helicity :

$$\lambda = \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}\,|} \Rightarrow \lambda = \pm 1$$



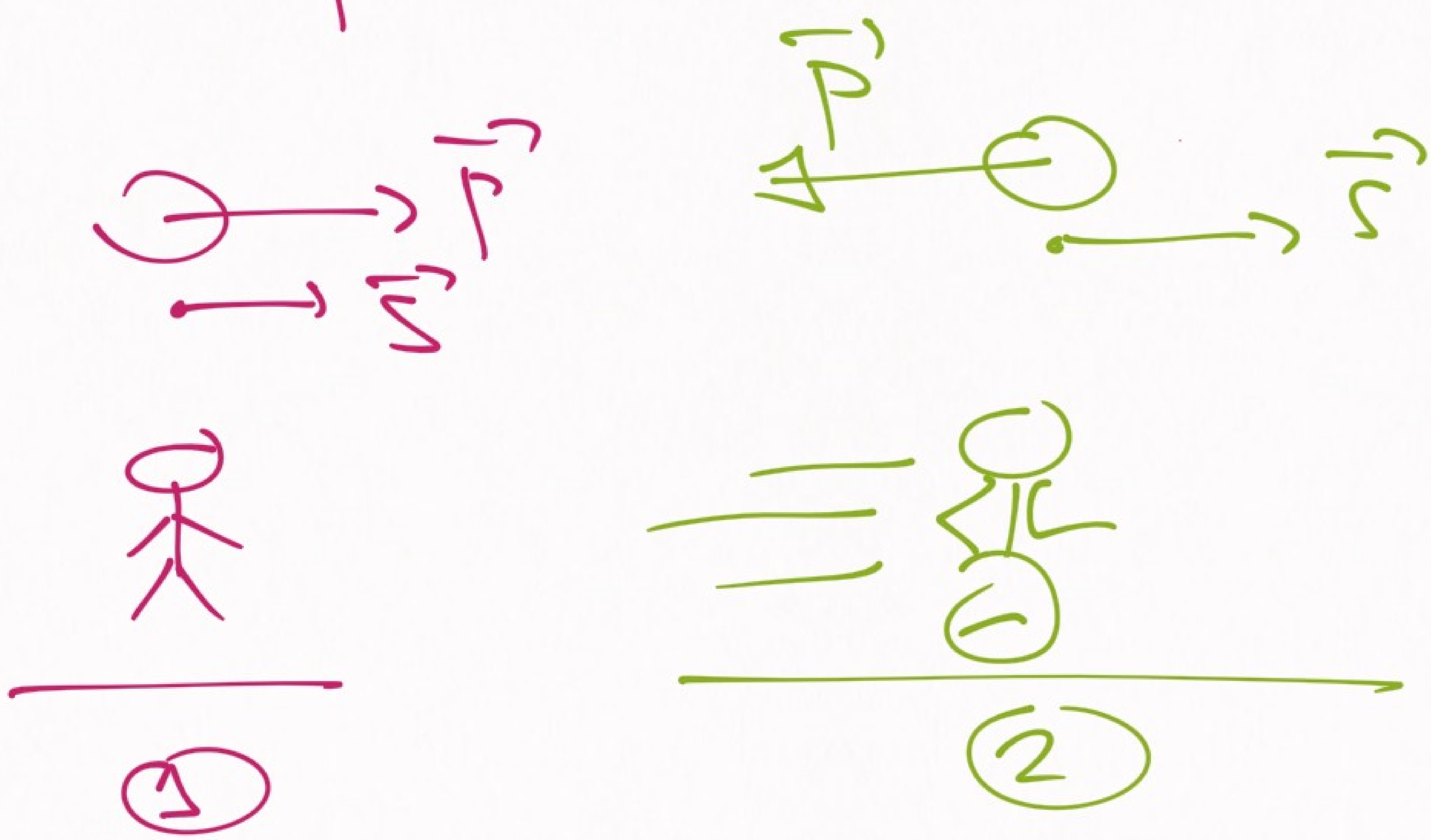


↳ a bit more beautiful like this

A detail:

1) massive particles ($|\vec{p}| < E$)

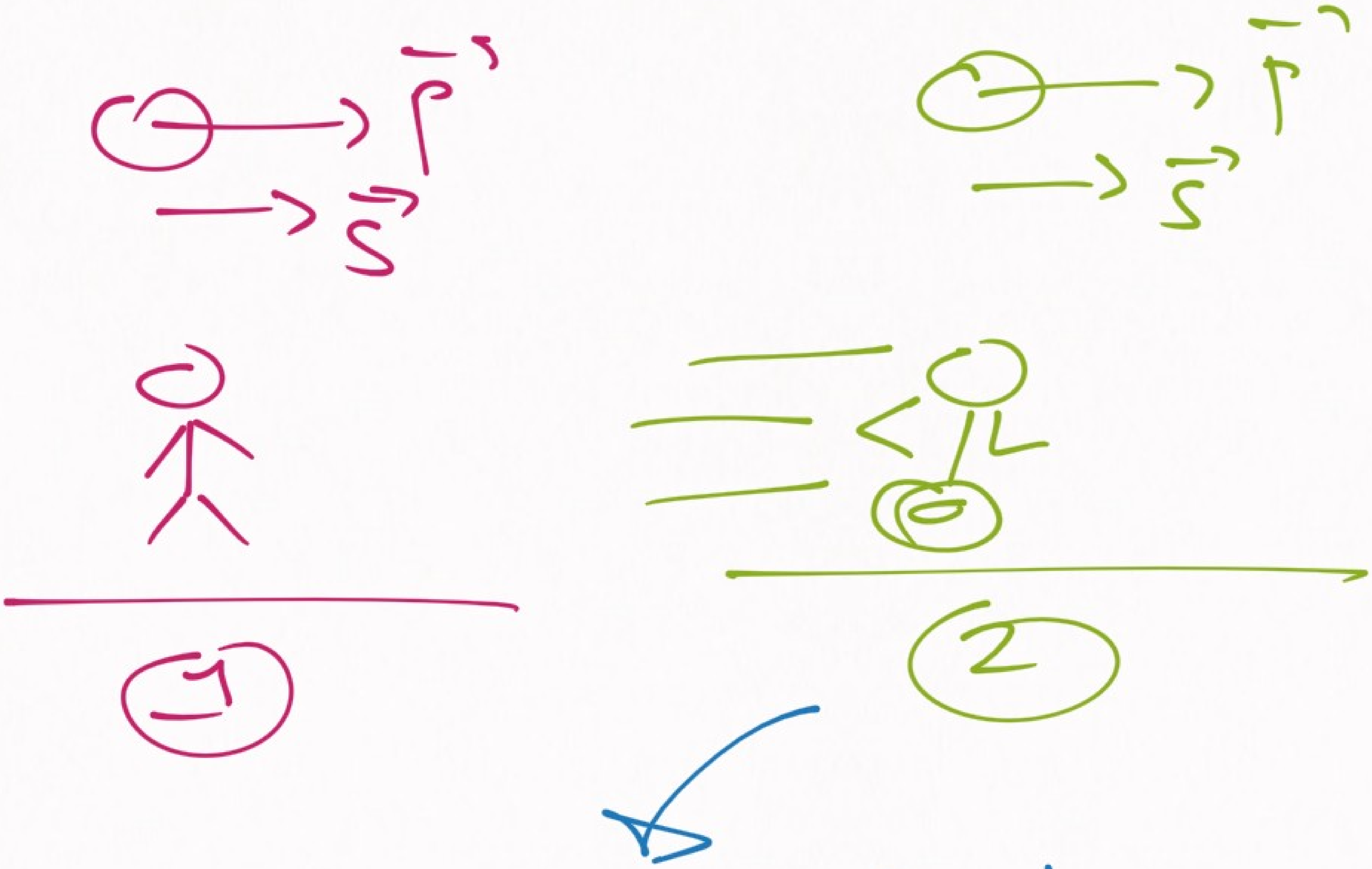
→ speed is relative



→ I can run fast enough as to change sign of the helicity

2) massless particle ($|\vec{p}| = E$)

$\rightarrow$ travels at light speed



no way you will outrun that massless particle

$\boxed{\text{Helicity conserved}}$

The QFT version of this:

$$\mathcal{L} = \underbrace{\bar{\psi}(i\not{\partial} - m)\psi}$$

mass

Dirac field

It has a symmetry:

$$\left[\begin{array}{l} \psi(x) \to e^{i\alpha}\psi(x) \\ \mathcal{L} \to \mathcal{L} \end{array}\right] \underline{\underline{U(1)}}$$

(or $U(1)_{L+R}$)

But we can also try this:

$$\psi_R = \frac{1}{2}(1+\gamma_5)\psi \;,\; \psi_L = \frac{1}{2}(1-\gamma_5)\psi$$

$$\left[\begin{array}{l} \psi_L(x) \to e^{i\alpha_L}\psi_L(x) \\ \psi_R(x) \to e^{i\alpha_R}\psi_R(x) \end{array}\right]$$

What happens w/ chiral transformations

1) $\bar{\psi}\not{\partial}\psi = \bar{\psi}_L\not{\partial}\psi_L + \bar{\psi}_R\not{\partial}\psi_R$

$[\bar{\psi}\not{\partial}\psi \to \bar{\psi}\not{\partial}\psi]$ ✓ CHECK!

2) $\bar{\psi}m\psi = \bar{\psi}_L m\psi_R + \bar{\psi}_R m\psi_L$

$$[\bar{\psi}m\psi \to e^{i(\alpha_R-\alpha_L)}\bar{\psi}_L m\psi_R + e^{-i(\alpha_R-\alpha_L)}\bar{\psi}_R m\psi_L]$$

✗ FAIL!

But if $m=0$, we have:

$$\boxed{\mathcal{L} \to \mathcal{L}}$$

✓ CHECK!

Now let's go to $\boxed{\text{QCD}}$

$$\mathcal{L} = \underbrace{-\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}}_{\text{gluons}} + \bar{q}\, i \not{D}\, q \underbrace{- \bar{q} \mathcal{M} q}_{\text{quark masses}}$$

$$q = \begin{pmatrix} u \\ d \\ s \\ c \\ b \\ t \end{pmatrix} \qquad \mathcal{M} = \begin{pmatrix} m_u & & 0 \\ & m_d & \\ 0 & & \ddots \end{pmatrix}$$

Question : $\boxed{\text{CAN WE IGNORE THE MASSES ?}}$

Answer : $\boxed{\text{only if } m_q < \Lambda_{QCD}}$

condition

Let's see ... $\Lambda_{QCD} \sim 200\text{-}300\,MeV$

$m_u \sim 2\,MeV$ ✓ CHECK!

$m_d \sim 5\,MeV$ ✓ CHECK!

$m_s \sim 95\,MeV$ ∼ MAYBE

$m_c \sim 1.3\,GeV$ ✗ 不行!

$m_b \sim 4.2\,GeV$ NOPE!

Two versions : $\underbrace{n = 2 \text{ or } n = 3}_{\#\text{ of light quarks}}$

CHIRAL TRANSFORMATIONS :

$$\boxed{\begin{gathered} q_L \to V_L q_L \,,\; q_R \to V_R q_R \\ q_L \in U(n)_L \,,\; q_R \in U(n)_R \end{gathered}}$$

$n = 2, 3$

SYMMETRY FOR HADRONS CONTAINING LIGHT QUARKS:

$$\boxed{G = U(n)_L \otimes U(n)_R}$$

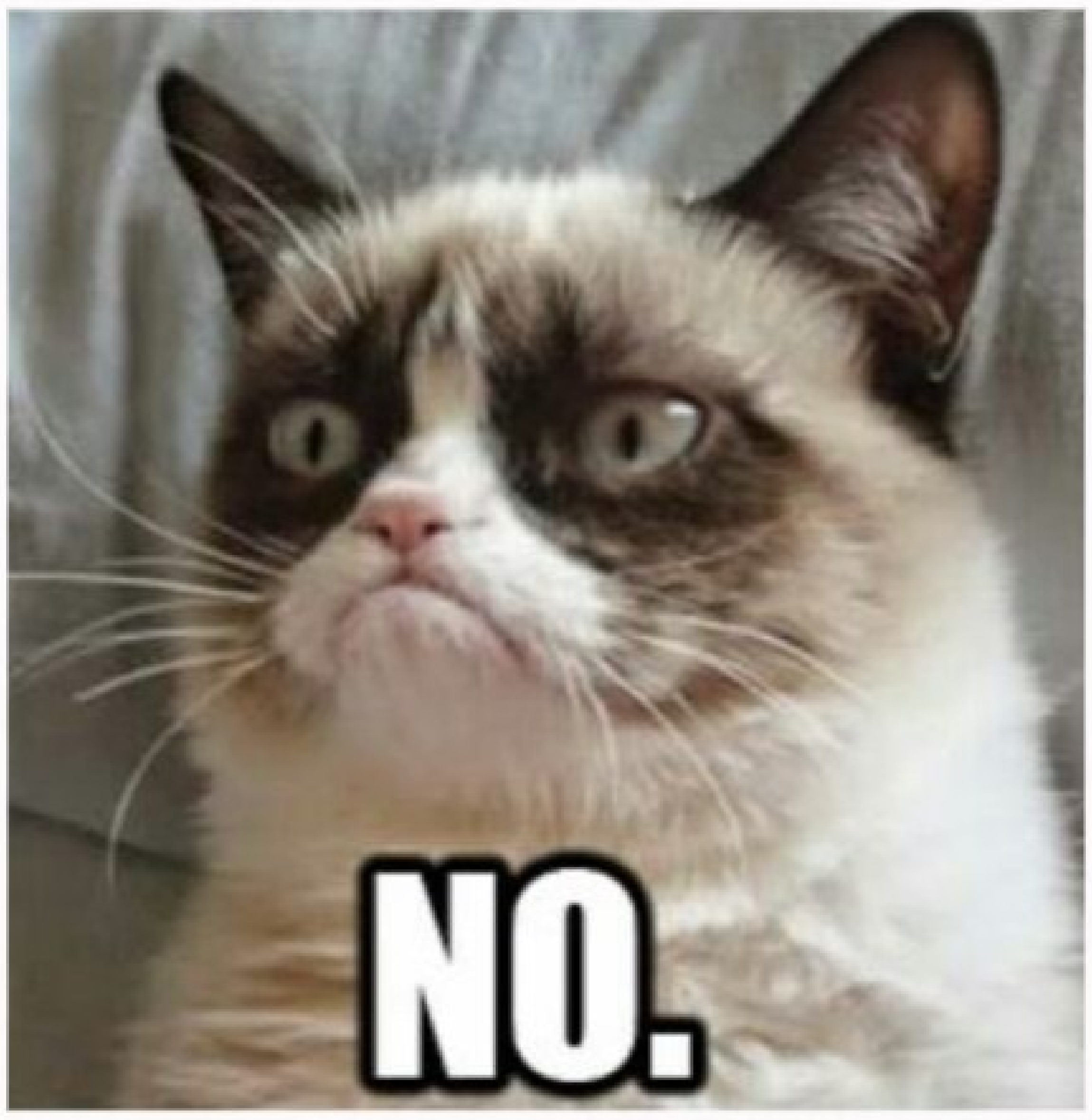
Really simple

$\Rightarrow$ Prediction of this symmetry:

$$\boxed{m(H, J^+) = m(H, J^-)}$$

$\rightarrow$ hadron masses do not depend on parity

DOES THIS HAPPENS IN NATURE?



$\rightarrow$ You heard Grumpy Cat!

Let's see what happens:

1) nucleon : $J = 1/2^+$, $m = 940$ MeV

$J = \frac{1}{2}^-$ "nucleon": $N(1535)$

$m = 1535$ MeV

2) ρ : $J = 1^-$, $m = 770$ MeV

$J = 1^+$ "ρ" : $b_1(1235)$

$m = 1230$ MeV

and the list goes on and on

$\Downarrow$

FUNDAMENTAL QCD STATE
NOT CHIRALLY SYMMETRIC

This looks like a job for the Goldstone theorem:

1) QCD chirally symmetric

$G = U(2)_L \otimes U(2)_R$

2) Lowest mass hadrons not chirally symmetric

$$\boxed{F = SU(2)_{L+R} \otimes U(1)_{L+R}}$$

$\Rightarrow \left[\frac{G}{F} \text{ has 3 generators} \right]$

$\hookrightarrow$ $\boxed{\text{3 massless bosons w/ } J^P = 0^-}$

$\hookrightarrow$ $\boxed{\text{THE PIONS}}$ ✓

Important: we have not explained the technical details

— ⊗ —

What's the big difference that chiral symmetry brings?

pions interactions are derivative

What does this mean?

pion interactions are weaker at low energies (or low momenta)

1) Before chiral symmetry

$$\mathcal{L} = i g_{\pi NN} \bar{\Psi}_N \gamma^5 \vec{\tau} \cdot \vec{\pi} \Psi_N$$

(old pion theories)

2) After chiral symmetry

$$\mathcal{L} = \frac{g_A}{2f_\pi} \bar{\psi}_N \gamma^5 \gamma_\mu \vec{\tau} \cdot \partial^\mu \vec{\pi} \psi_N$$

[derivative interaction]

Comparing 1 + 2)

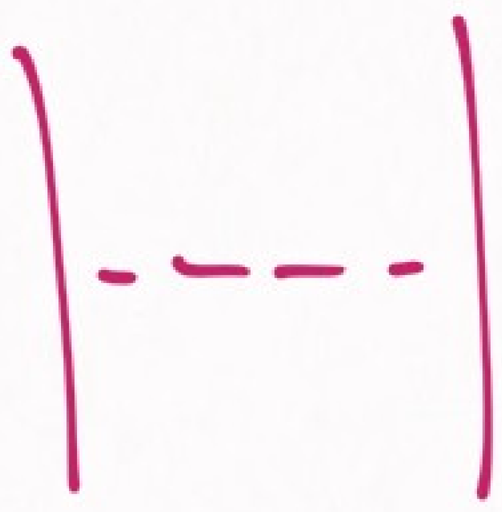
$$\boxed{\frac{g_A}{f_\pi} = \frac{g_{\pi NN}}{M_N}}$$

Goldberger-Treimann relation

$g_A \sim 1.26$, $f_\pi \sim 92\,\mathrm{MeV}$, $g_{\pi NN} \sim 13$

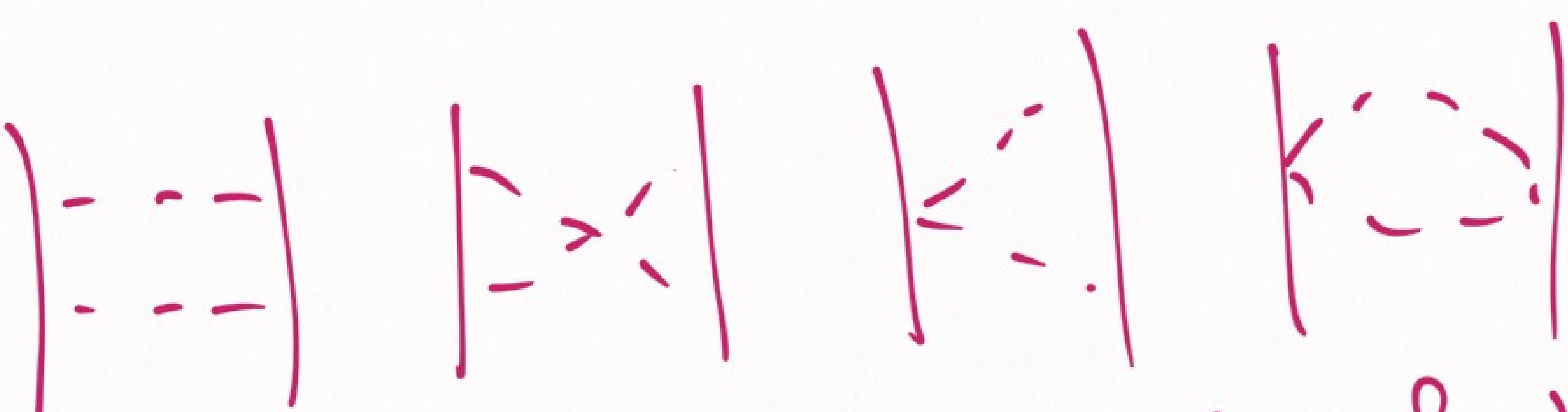
Derivative pion interaction:

1) OPE → no difference



→ this diagram is identical for non-derivative / derivative pions

2) TPE → big difference



planar box | crossed box | triangle | football

Derivative pions fundamental for getting TPE right!

Next lesson → review of two-body

1) Easy introduction:

4. **Chiral effective Lagrangians**
H. Leutwyler (Bern U.). Jul 1991. 55 pp.
Published in **Lect.Notes Phys. 396 (1991) 1-37**
BUTP-91-26
DOI: 10.1007/3-540-54978-1_8
Conference: C91-06-02, p.97-138, Conference: C91-02-27
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
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2) Classical review:

1. (1099) **Chiral dynamics in nucleons and nuclei**
V. Bernard (Strasbourg, CRN), Norbert Kaiser (Munich, Tech. U.), Ulf-G. Meissner (Bonn U.).
Jan 1995. 153 pp.
Published in **Int.J.Mod.Phys. E4 (1995) 193-346**
CRN-95-3, TK-95-1
DOI: 10.1142/S0218301395000092
e-Print: **hep-ph/9501384** | **PDF**
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