Nuclear Physics 10

Isospin & SU(3)-flavor symmetries

ISOSPIN SYMMETRY

$p, n \rightarrow M_p = 938.272$ MeV
$M_n = 939.563$ MeV

$\Rightarrow M_p \simeq M_n$

Usually we treat them as one particle w/ two states:

$\boxed{\text{THE NUCLEON}}$

$N = \begin{pmatrix} p \\ n \end{pmatrix}_I \rightarrow$ analogous to spin (isospin)

$\Rightarrow |I\,M_I\rangle_I$ $\begin{cases} |\frac{1}{2}\,\frac{1}{2}\rangle_I = |p\rangle \\ |\frac{1}{2}\,-\frac{1}{2}\rangle_I = |n\rangle \end{cases}$

Isospin algebra → identical to spin algebra

Two nucleon system:

$|NN\rangle_I \rightarrow \frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$

$\boxed{I=0}$ (Isoscalar)

$$|00\rangle_I = \frac{1}{\sqrt{2}}\left[|+-\rangle_I - |-+\rangle_I\right]$$

$$= \frac{1}{\sqrt{2}}\left[|pn\rangle - |np\rangle\right]$$

$\boxed{I=1}$ (Isovector)

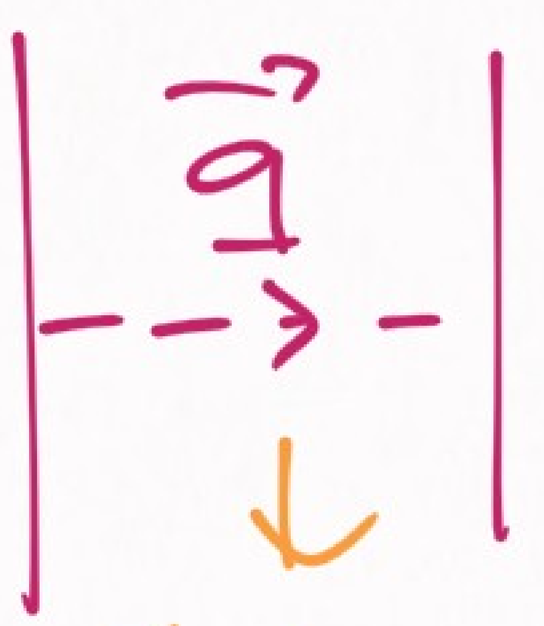
$$|11\rangle_I = |pp\rangle$$

$$|10\rangle_I = \frac{1}{\sqrt{2}}\left[|pn\rangle + |np\rangle\right]$$

$$|1-1\rangle_I = |nn\rangle$$

We can go back to OPE :

1) Before isospin (LECTURE 4)

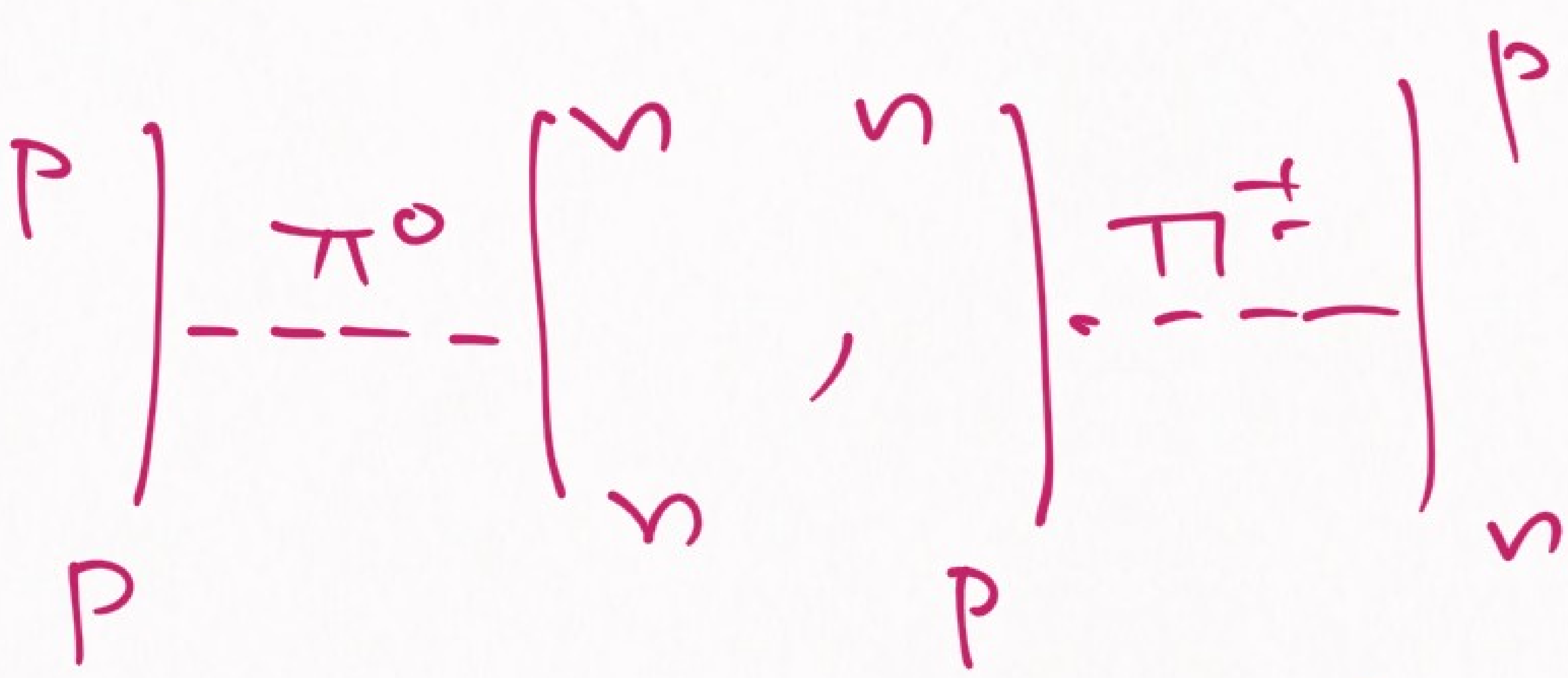


$$= -\left(\frac{g_{\pi NN}}{2M}\right)^2 \frac{\vec{\sigma}_1\cdot\vec{q}\;\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^{\,2}+m_\pi^2}$$

$$1/\vec{q}^{\,2}+m_\pi^2$$

$$\frac{g_{\pi NN}}{2M}\,\vec{\sigma}_1\cdot\vec{q} \qquad\qquad \frac{g_{\pi NN}}{2M}\,\vec{\sigma}_1\cdot(-\vec{q})$$

→ then we said that we have to multiply by (−3) to get the deuteron right:

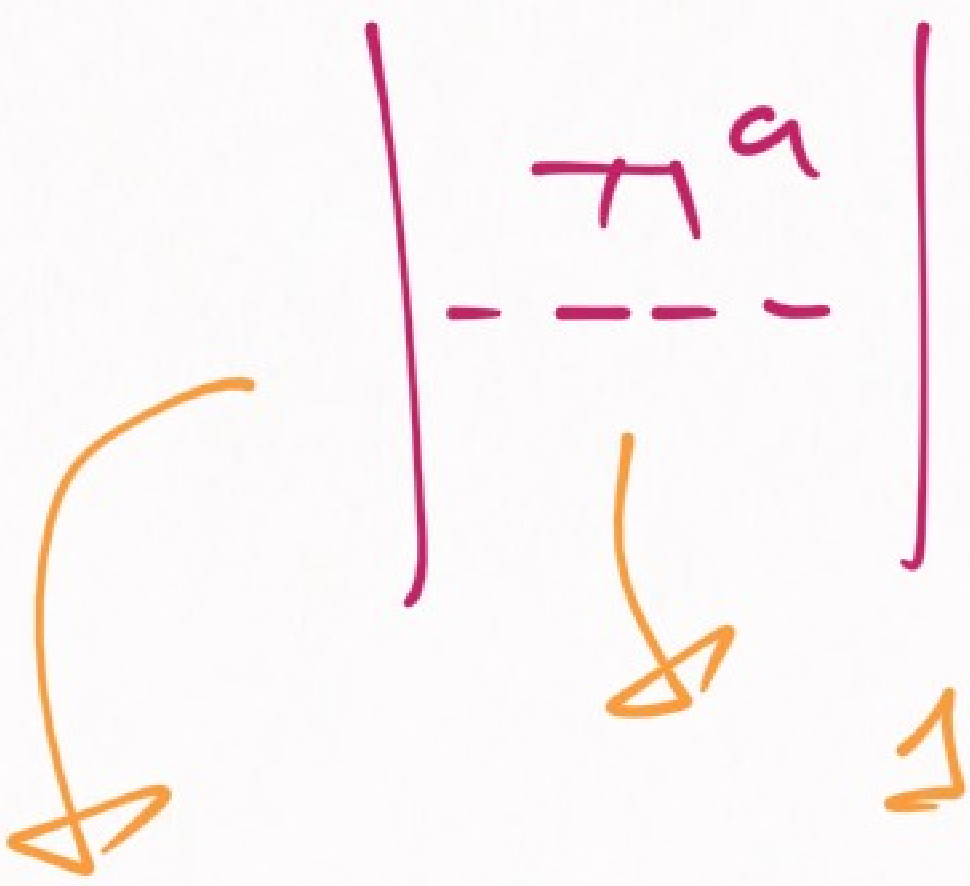


$$\Rightarrow \begin{pmatrix} -1 & +2 \\ +2 & -1 \end{pmatrix} \longrightarrow$$

eigenvalues

$$\boxed{1, -3}$$

2) With isospin



$$\rightarrow \left(\frac{g_{\pi NN}}{2M_N}\vec{\sigma}_2\cdot(-\vec{q})\times\tau_2^{a\dagger}\right)$$

$$1/(\vec{q}^2+m^2)$$

$$\left(\frac{g_{\pi NN}}{2M_N}\vec{\sigma}_1\cdot\vec{q}\times\tau_1^a\right)$$

[The pion is a matrix in isospin space]

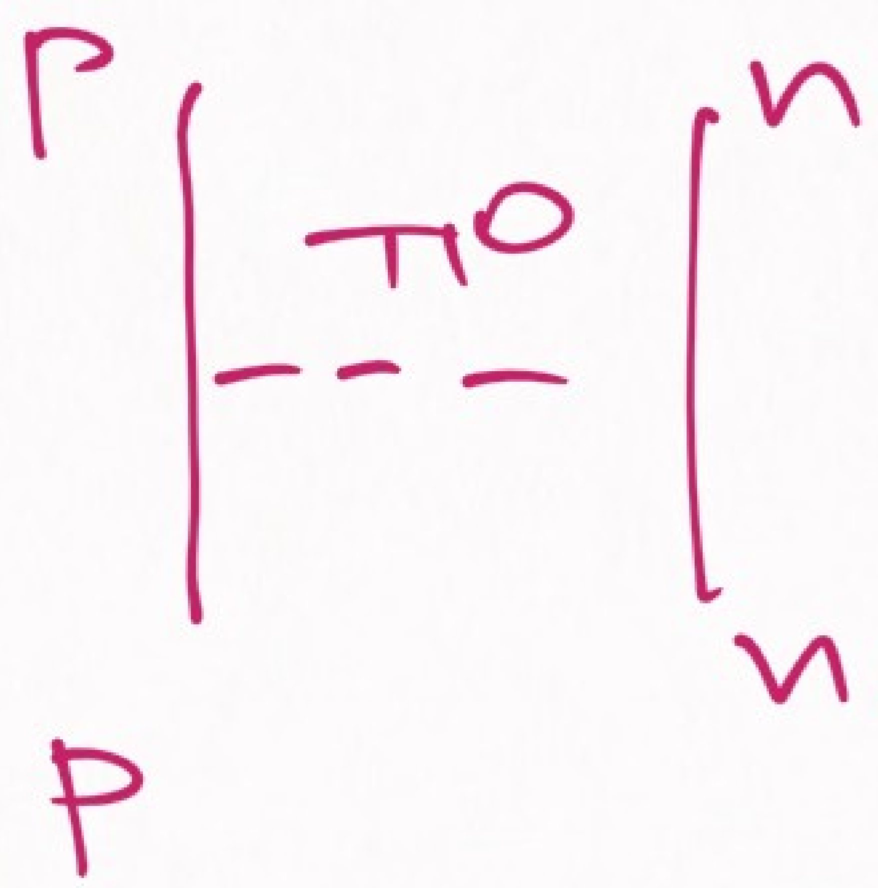
$$\pi^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(chabuduo, didn't check sign of π^+ & π^-)

$$\pi^+ = \begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix}$$

$$\pi^- = \begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \end{pmatrix}$$

This gives us extra factors



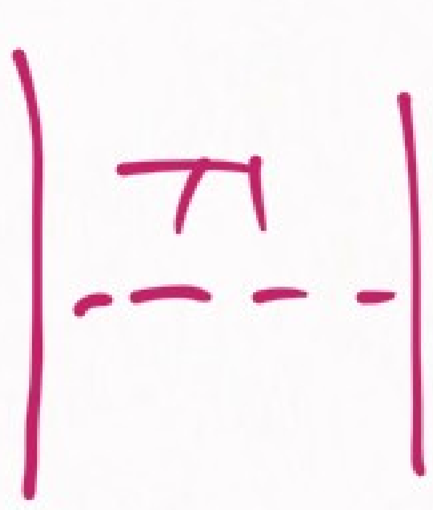
factors:

$\pi^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

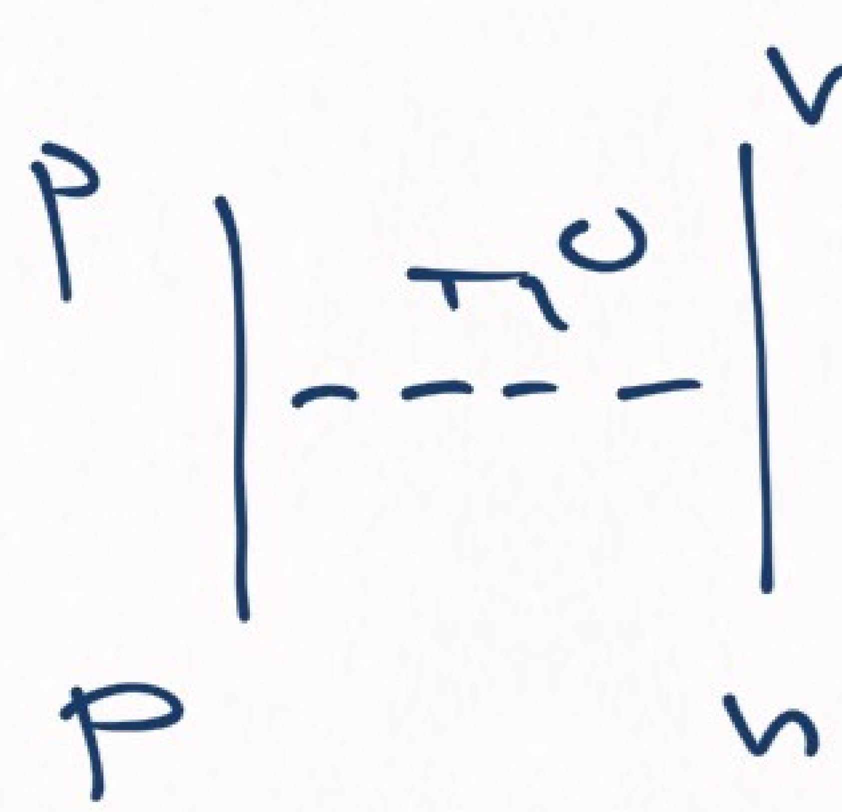
$\langle p|\pi^0|p\rangle = +1$ $\langle n|\pi^0|n\rangle = -1$

$(\langle p|\pi^0|n\rangle = 0, \langle n|\pi^0|p\rangle = 0)$

NO ISOSPIN $\Rightarrow$ ISOSPIN



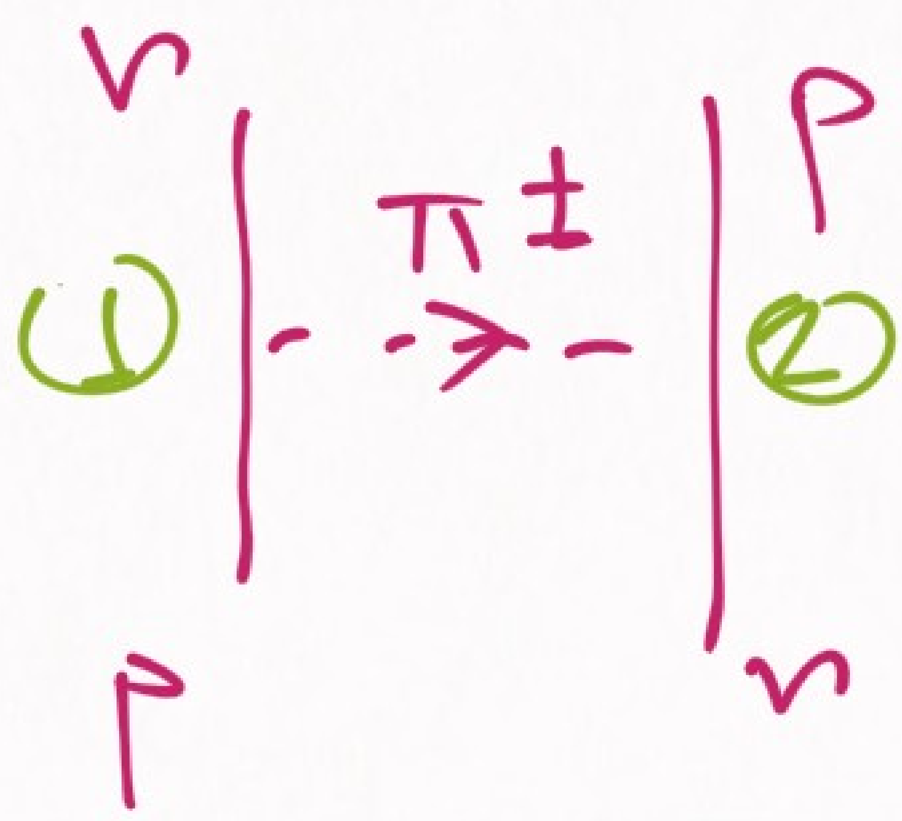
$= V_0$



$= -V_0$

a factor of (-1) for π^0

For the charged pion:



Factors:

$\pi^+ = \begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix}$, $\pi^- = \begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \end{pmatrix}$

↳ it's a virtual particle:

if in (1) is $\pi^+ \Rightarrow$ in (2) is π^-

[(1) is $\pi^- \Rightarrow$ in (2) is π^+]

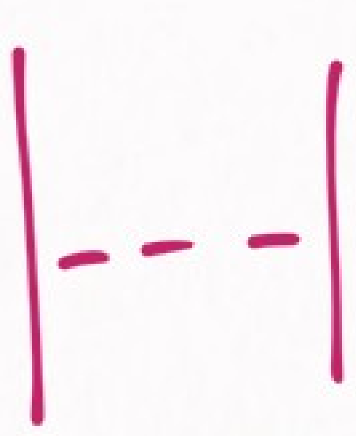
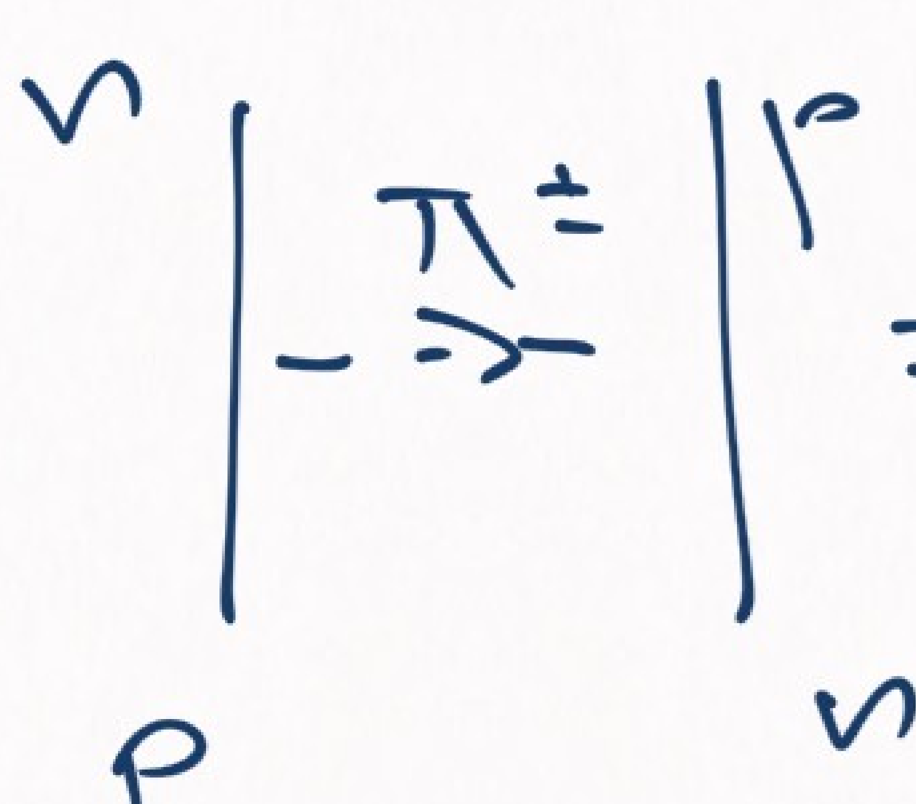
↓

(*)

(1) $\rightarrow \langle n | \pi^+ | p \rangle = \sqrt{2}$

(2) $\rightarrow \langle p | \pi^- | n \rangle = \sqrt{2}$

No ISOSPIN $\Rightarrow$ ISOSPIN

 $= V_0 \rightarrow$ 

$= 2V_0$

(*) $\rightarrow$ π^+ forward in time $\triangleq$ π^- backwards in time

Putting the pieces together:

$$\mathcal{B} = \{ |pn\rangle, |np\rangle \}$$

$$\left| \text{---} \right| = V_0 \Rightarrow \left| \text{---} \right| = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix} V_0$$

Eigenvalues: $\lambda = -3, 1$

Eigenvectors:

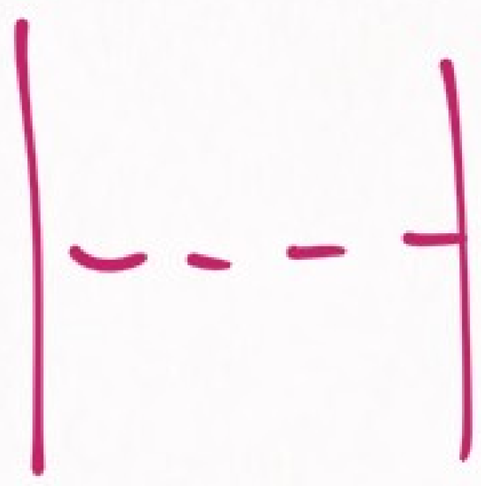
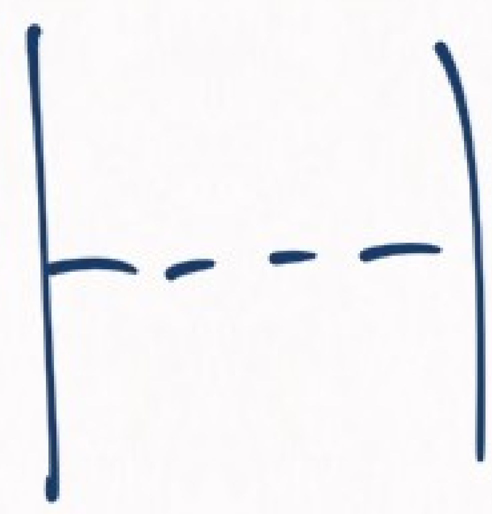
$$|-3\rangle = \frac{1}{\sqrt{2}} \{ |pn\rangle - |np\rangle \}$$

$$|1\rangle = \frac{1}{\sqrt{2}} \{ |pn\rangle + |np\rangle \}$$

$$\mathcal{B} = \{ |pp\rangle, |nn\rangle \}$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} V_0$$

But normally we simply use the isospin basis:

 $= V_0 \Rightarrow$  $= \vec{\tau}_1 \cdot \vec{\tau}_2 \, V_0$

$\rightarrow$ [Exactly like spin]

$$\vec{\tau}_1 \cdot \vec{\tau}_2 = \begin{cases} -3 & \text{for } I = 0 \\ +1 & \text{for } I = 1 \end{cases}$$

$$(\vec{I} = \vec{I}_1 + \vec{I}_2)$$

DEUTERON $\Rightarrow I = 0$

$$|d\rangle = \frac{1}{\sqrt{2}}\left[|pn\rangle + |np\rangle\right]$$

This extends to the OBE:

1) $\pi, \rho \to$ isospin-1

$V_0 \longrightarrow \bar{\tau}_1 \cdot \bar{\tau}_2 V_0$ (change)

2) $\sigma, \omega \to$ isospin-0

$V_0 \longrightarrow V_0$ (no change)

$\longleftarrow \otimes \longrightarrow$

RELATED CONCEPT:

[Extended Dirac-Fermi statistics]

$p, n \to$ fermions

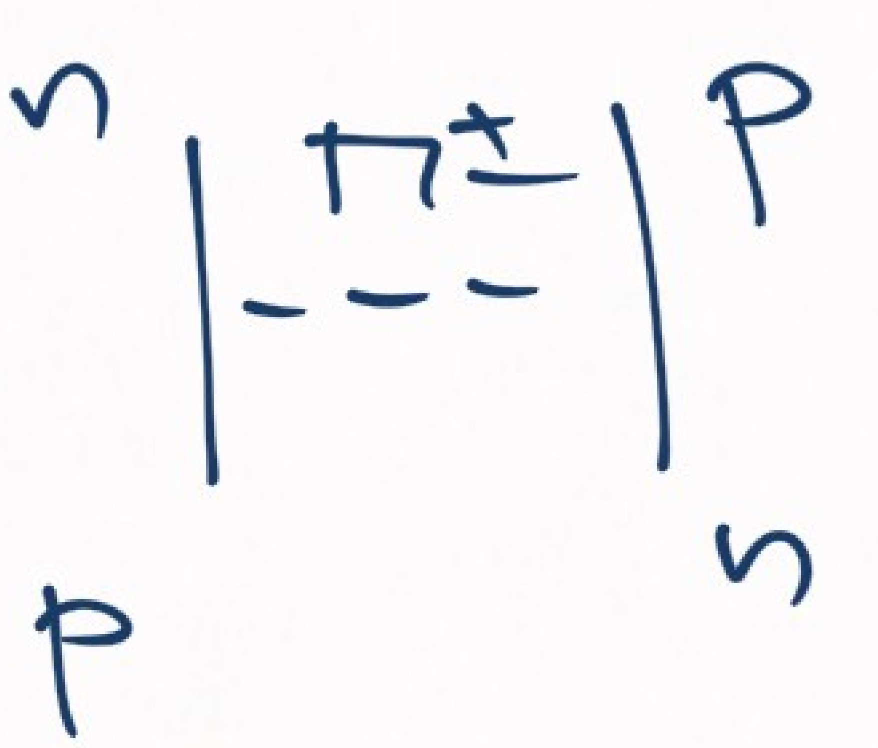
pp system $\to$ antisymmetric

nn system $\to$ antisymmetric

But in principle:

$np \to$ different particles, no symmetry

However, if the potential can exchange $(p \leftrightarrow n)$, this is no longer true



$\Rightarrow$

$p \to n$

$n \to p$

We extend the (anti)symmetry:

$N = \begin{pmatrix} p \\ n \end{pmatrix}$, $NN \to$ antisymmetric

Practical implications:

NN $\longrightarrow$ spatial : $(-1)^{L}$

spin : $(-1)^{S+1}$

isospin : $(-1)^{I+1}$

$\Rightarrow (-1)^{L+S+I} = -1$

$\Rightarrow$ $L+S+I$ must be odd

DEUTERON:

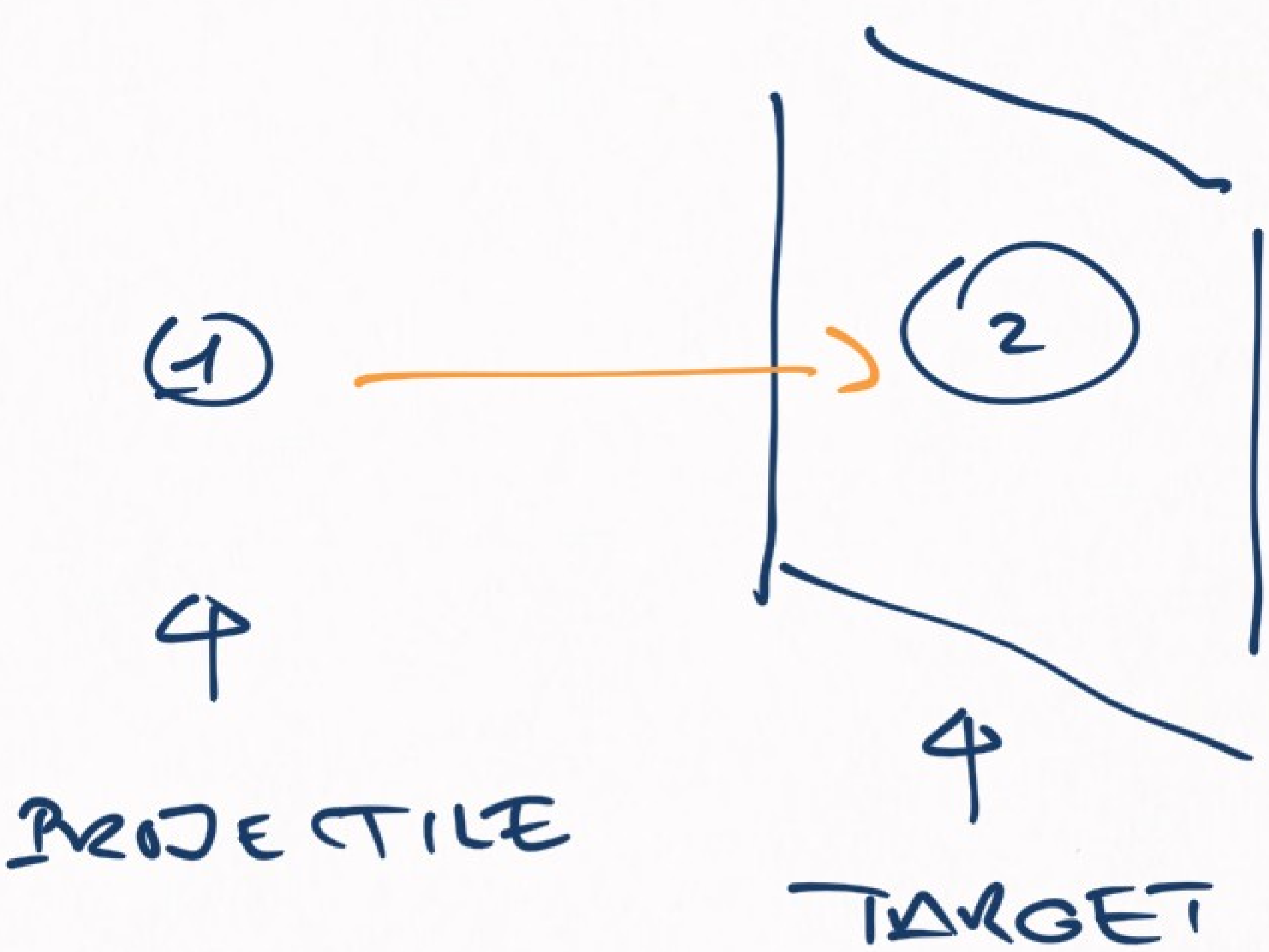
$$\boxed{L=0, I=0} \Rightarrow \boxed{S=1}$$

$\rightarrow$ [The deuteron has to have spin-1]

A few tests of isospin symmetry

→ pp, np, nn $\underbrace{\text{scattering length}}_{\text{what is this?}}$

Scattering → cross section



$\boxed{\text{CLASSICAL}}$

$$\underbrace{\sigma = \pi (R_1 + R_2)^2}_{\text{cross section}}$$

(≃ effective area of target)

$\boxed{\text{QUANTUM}}$

$$\sigma \xrightarrow[E \to 0]{} 4\pi |a_0|^2$$

↳ scattering length

Let's see :

$$a_0^s(pp) \simeq -17.3 \text{ fm}$$
$$a_0(np) \simeq -23.7 \text{ fm}$$
$$a_0(nn) \simeq -18.6 \text{ fm}$$

LARGE (UNNATURAL) & SIMILAR

But it's not perfect :

1) CSB (Charge symmetry breaking)

$$a_0^s(pp) \neq a_0(nn)$$

2) CIB (Charge independence breaking)

$$a_0(np) \neq \frac{1}{2}\left[a_0^s(pp) + a_0(nn)\right]$$

→ the breaking is small though

→ $\boxed{\text{ISOSPIN WORKS WELL}}$

More examples:

1) ^3H & ^3He

$$|{}^3He\rangle = |\tfrac{1}{2}\,+\tfrac{1}{2}\rangle_I \quad (ppn)$$

$$|{}^3H\rangle = |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle_I \quad (pnn)$$

$$B({}^3He) = 7.72 \text{ MeV}$$

$$B({}^3H) = 8.48 \text{ MeV}$$

$$\Rightarrow B({}^3He) \simeq B({}^3H) \checkmark$$

2) ^6He, ^6Li, ^6Be

$$(|1\,-1\rangle_I,\ |1\,0\rangle_I,\ |1\,+1\rangle_I)$$

$$B({}^6He) = 29.27 \text{ MeV}$$

$$B({}^6Li) = 31.99 \text{ MeV}$$

$$B({}^6Be) = 26.92 \text{ MeV}$$

3) Pion masses:

$$m(\pi^0) = 134.977 \text{ MeV}$$
$$m(\pi^\pm) = 139.570 \text{ MeV}$$

[ISOSPIN SYMMETRY WORKS REALLY WELL]

1) Isospin symmetry 的源头:

$$m_u, m_d \ll \Lambda_{QCD}$$

2) But the strange quark is also light:

$$m_s \lesssim \Lambda_{QCD}$$

Maybe we can extend isospin to the strange quark

$\Rightarrow$ $\boxed{\text{SU(3)-FLAVOR SYMMETRY}}$

1) Isospin $\rightarrow$ SU(2) Symmetry

2) Flavor $\rightarrow$ SU(3) Symmetry

[SU(N) are N×N matrices / $U^{\dagger}U = 1$, $\det U = 1$]

Flavoor $\rightarrow$ n, p have partners

$\rightarrow$ $\boxed{\text{BARYON OCTET}}$

$\leftarrow$ (UNIVERSITY OF MANCHESTER)

$\boxed{SU(3)_F}$

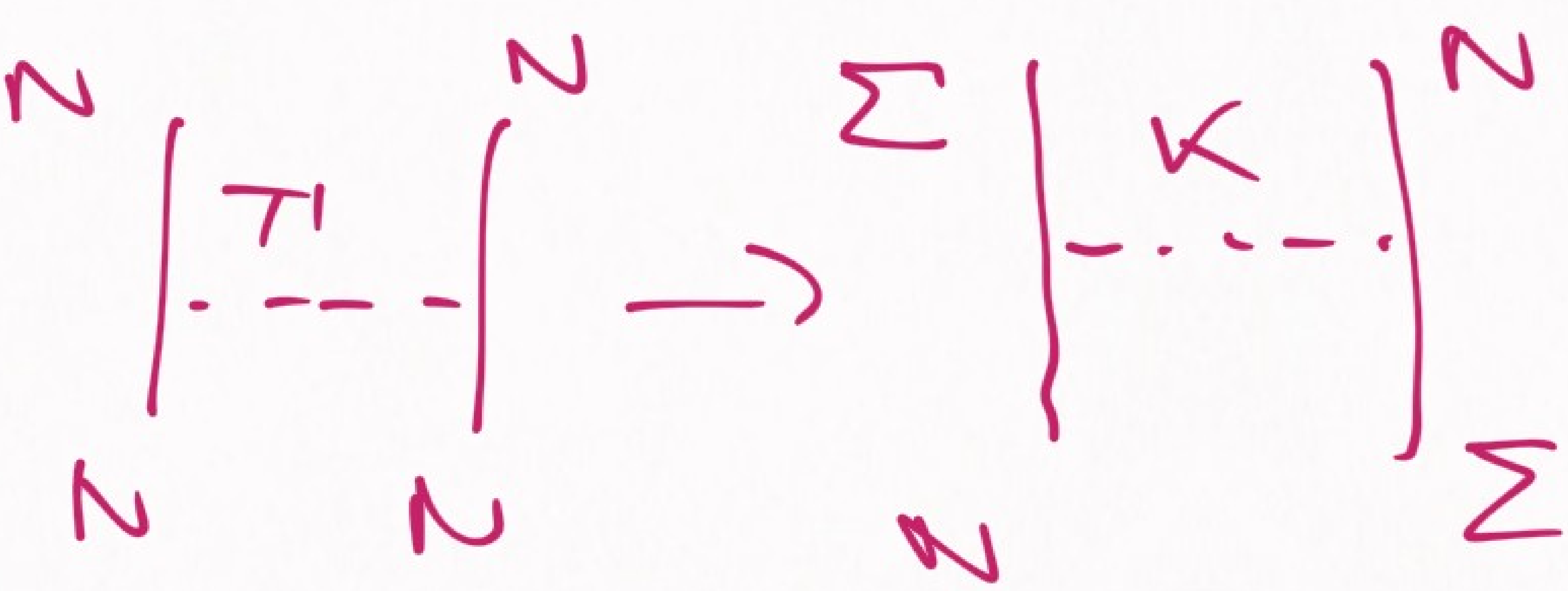
1) Works well for couplings

2) Works a bit less well for masses

— ⊗ —

1) Hyperon-Hyperon equivalent of OPE :

Hyperon → baryon w/ strange quarks

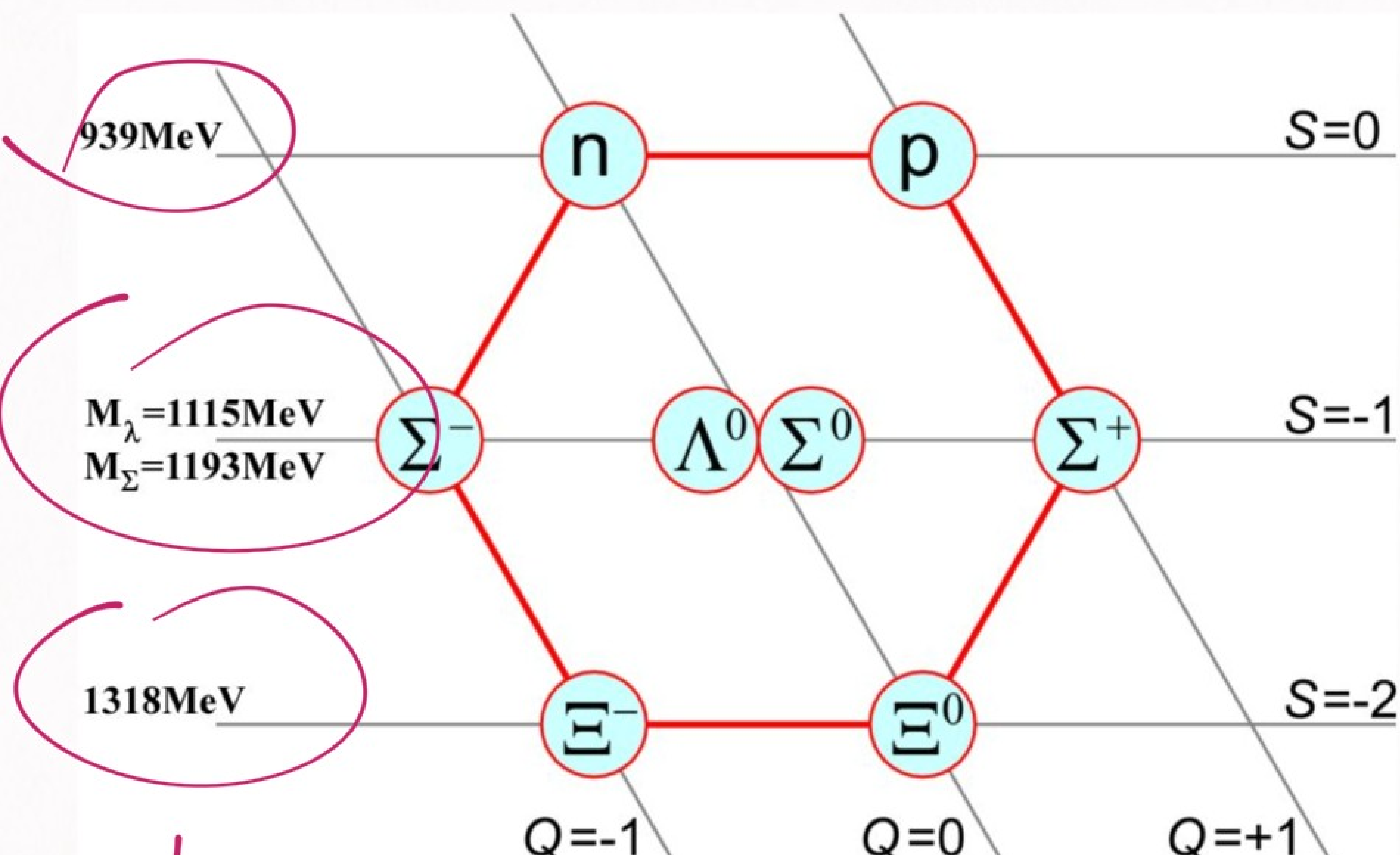


$$\bar{\tau}_1 \cdot \bar{\Sigma}_2 V_0 \longrightarrow \frac{1}{2}\left[1 + \bar{\tau}_1 \cdot \bar{T}_2\right] V_0$$

Σ has isospin $I = 1$

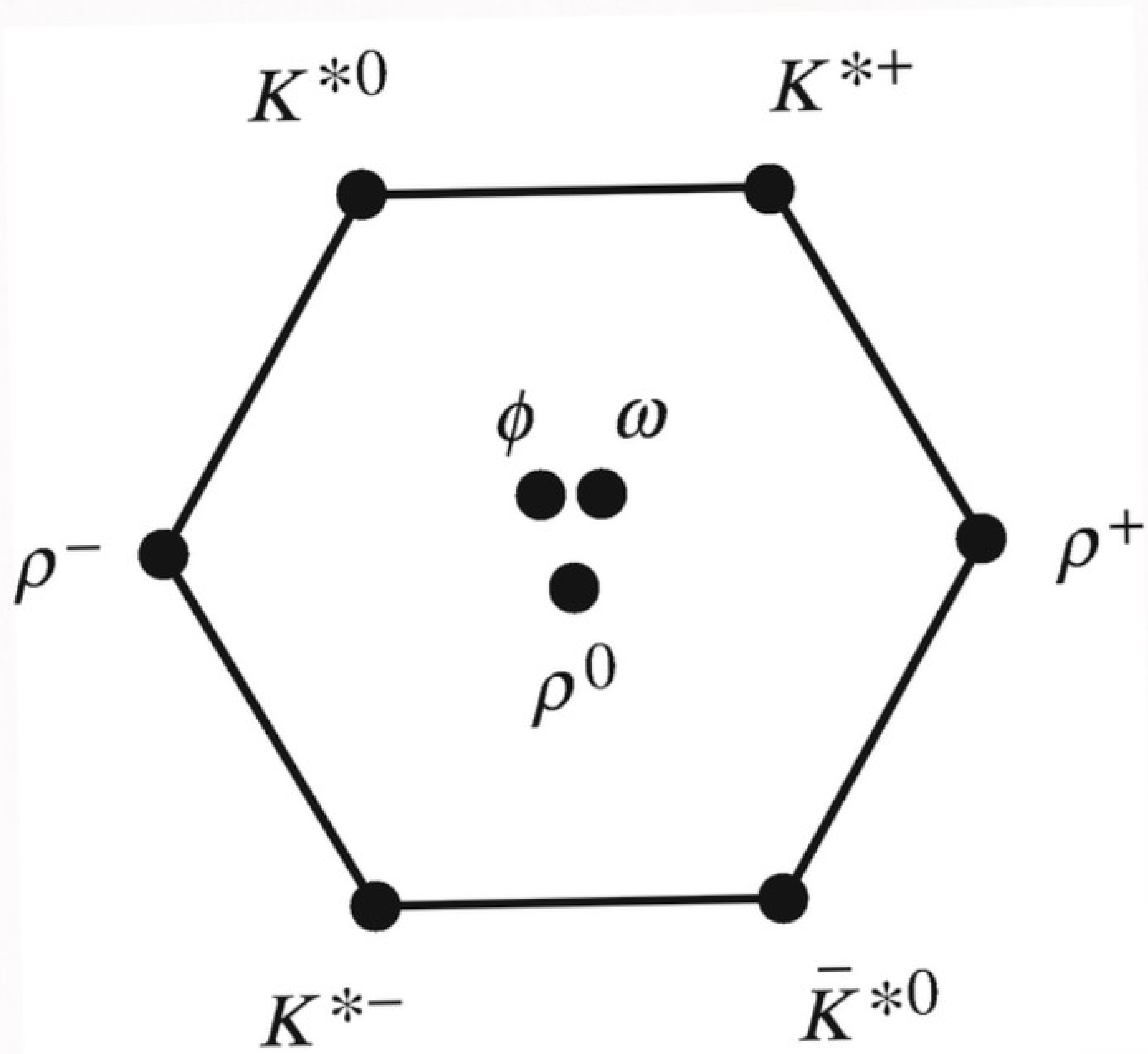
→ But we will not explain the details here

2) Masses → $(\underbrace{m_s \gg m_u, m_d}_{\text{mismatches}})$



↳ masses of the baryon octet similar, but not that much

Vector meson nonet:



$m_\rho \simeq 770$ MeV

$m_\omega \simeq 780$ MeV

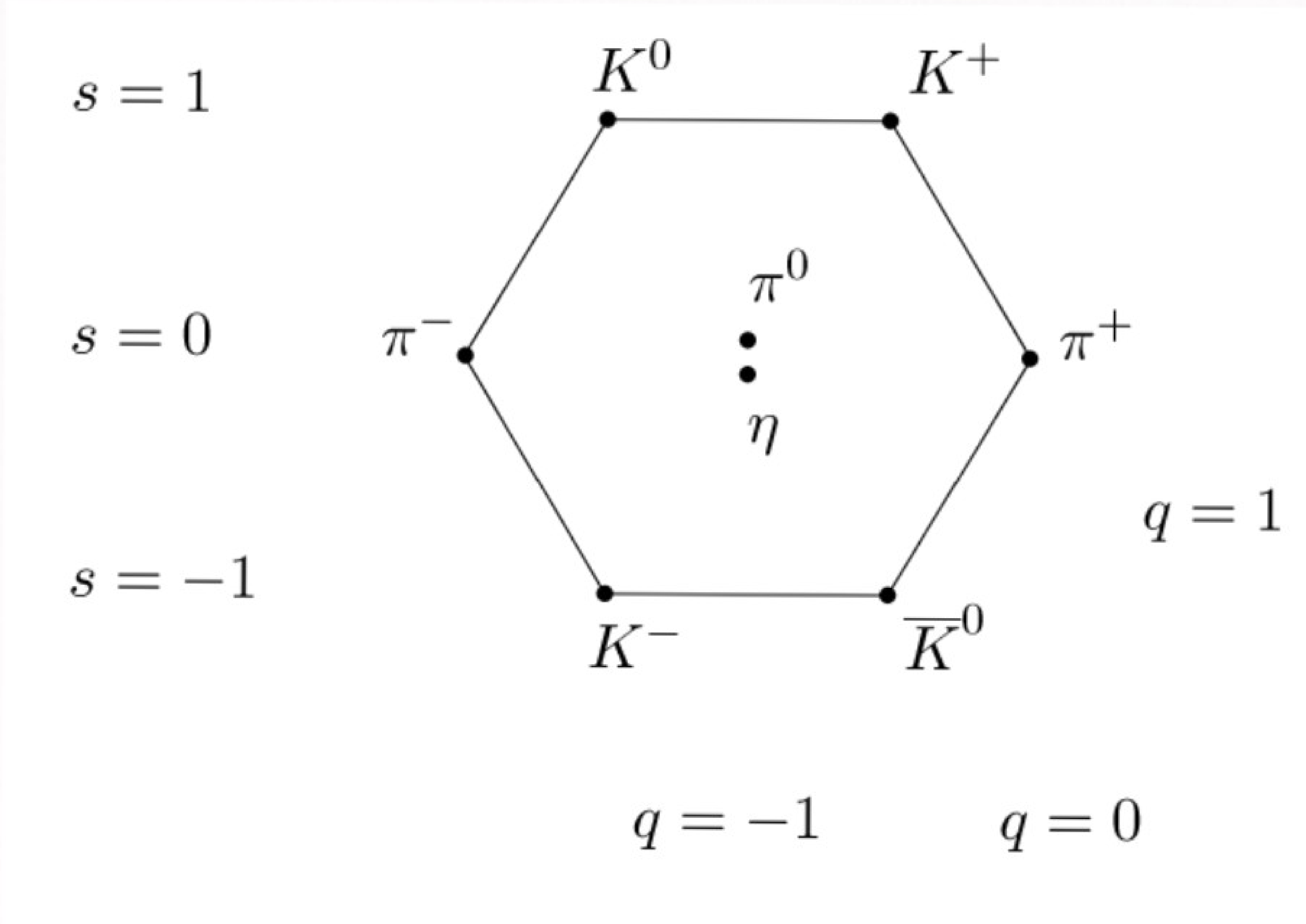
Isospin symmetry

$m_{K^*} \simeq 890$ MeV

$m_\phi \simeq 1020$ MeV

$\to$ SU(3)-flavour $\Rightarrow$ not ideal, but not that bad either

Pseudoscalar meson octet:



$$\left.\begin{array}{l} m_\pi \simeq 138\ \text{MeV} \\ m_K \simeq 495\ \text{MeV} \\ m_\eta \simeq 548\ \text{MeV} \end{array}\right\}\ \text{OUPS!}$$

→ This is batshit crazy

What's going on here?

→ [CHIRAL SYMMETRY]

NEXT LESSON:

→ linear sigma model
& Goldstone theorem

→ Chiral symmetry

Is it next monday or
next thursday?

SU(3)-flavor references:

→ "Quarks & Leptons" Ch. 2
(Halzen & Martin)
Really good reference
→ very much recommended

→ The original papers:

Symmetries of baryons and mesons

[2] Murray Gell-Mann (Caltech)
Phys.Rev. 125 (1962) 1067-1084 • DOI: 10.1103/PhysRev.125.1067 •
http://tuvalu.santafe.edu/~mgm/Site/Publications_files/MGM%2037.pdf

Derivation of strong interactions from a gauge invariance

[3] Yuval Ne'eman (Imperial Coll., London)
Nucl.Phys. 26 (1961) 222-229 • DOI: 10.1016/0029-5582(61)90134-1

→ The indispensable reference on SU(3) Clebsch-Gordans:

← rpp2017-rev-su3-isoscalar-factors.pdf

46. SU(3) Isoscalar Factors and Representation Matrices

Written by R.L. Kelly (LBNL).

The most commonly used SU(3) isoscalar factors, corresponding to the singlet, octet, and decuplet content of $8 \otimes 8$ and $10 \otimes 8$, are shown at the right. The notation uses particle names to identify the coefficients, so that the pattern of relative couplings may be seen at a glance. We illustrate the use of the coefficients below. See J.J de Swart, Rev. Mod. Phys. **35**, 916 (1963) for detailed explanations and phase conventions.

A $\sqrt{\ }$ is to be understood over every integer in the matrices; the exponent 1/2 on each matrix is a reminder of this. For example, the $\Xi \to \Omega K$ element of the $10 \to 10 \otimes 8$ matrix is $-\sqrt{6}/\sqrt{24} = -1/2$.

Intramultiplet relative decay strengths may be read directly from the matrices. For example, in decuplet → octet + octet decays, the ratio of $\Omega^* \to \Xi\overline{K}$ and $\Delta \to N\pi$ partial widths is, from the $10 \to 8 \times 8$ matrix,

$$\frac{\Gamma\left(\Omega^* \to \Xi\overline{K}\right)}{\Gamma\left(\Delta \to N\pi\right)} = \frac{12}{6} \times \text{(phase space factors)}\ . \tag{46.1}$$

Including isospin Clebsch-Gordan coefficients, we obtain, *e.g.*,

$$\frac{\Gamma(\Omega^{*-} \to \Xi^0 K^-)}{\Gamma(\Delta^+ \to p\pi^0)} = \frac{1/2}{2/3} \times \frac{12}{6} \times p.s.f. = \frac{3}{2} \times p.s.f. \tag{46.2}$$

(Check it on pdglive or in the 企业 wechat files)

→ the previous is based on

The Octet model and its Clebsch-Gordan coefficients

J.J. de Swart (CERN) (1963)

Published in: *Rev.Mod.Phys.* 35 (1963) 916-939, *Rev.Mod.Phys.* 37 (1965) 326-326 (erratur

DOI cite

Anecdote: once upon a time, de Swart & Machleidt ended up fighting (for real) in a workshop

(physics requires passion)

→ extremely useful reference:

Tables of SU(3) isoscalar factors

Thomas A. Kaeding (LBL, Berkeley)

Atom.Data Nucl.Data Tabl. 61 (1995) 233-288 • *e-Print: nucl-th/9502037* • *DOI: 10.1006/adnd.1995.1011*