Nuclear Physics 9

RECAP: THE EFT RECIPE

1) Consider some system

1.a) "True potential" not necessary

1.b) Only need the scales (size of bound state, range of interaction)

2) Consider an effective interaction

$$V_C(\vec{r}) = C_0 \delta^{(3)}(\vec{r}), \quad V_C(\vec{q}) = C_0$$

3) Regularize & Renormalize

$$V_C(\vec{q}) = C_0 \Rightarrow V_C(\vec{q}) = C_0(\Lambda) e^{-q^2/\Lambda^2}$$

$\Rightarrow$ From $\frac{d}{d\Lambda}\langle \Psi | H | \Psi \rangle = 0$

we compute $C_0(\Lambda)$

4) Check that the system is indeed correctly described

Many formulations of this idea

Example: this one by D.R. Phillips

EFT with NDA: the algorithm

1. Identify the relevant degrees of freedom
2. Identify high- and low-energy scales→expansion parameters x
3. Identify symmetries of low-energy theory
4. Choose the accuracy required. This, together with the size of x, tells you the order, n, to which you must calculate.
5. Write down all possible local operators, that have naive dimensions up that order, and are consistent with symmetries

"NDA"

6. Derive the behaviour of loops, and calculate them.

All operators needed for renormalization at this order should be present →Model independence

By now, this should sound familiar

and now that we are talking about different ways to consider a problem..

The multiple ways of looking at things

Gals & Guys,

I Know you watch anime...

(your laoshi does it too)

Let's remember the six paths of Pain:

(those sweet memories of when I was a postdoc in Jülich)

Oh wait, not of Pain...

The six paths of Feynman!

Feynman Rikudo

费曼六道

- Therefore psychologically we must keep all the theories in our heads, and every theoretical physicist who is any good knows six or seven different theoretical representations for exactly the same physics.
  - chapter 7, “Seeking New Laws,” p. 168

对于同一个理论，

只要是优秀的物理学家，

都知道六，七种不同的

表示方法

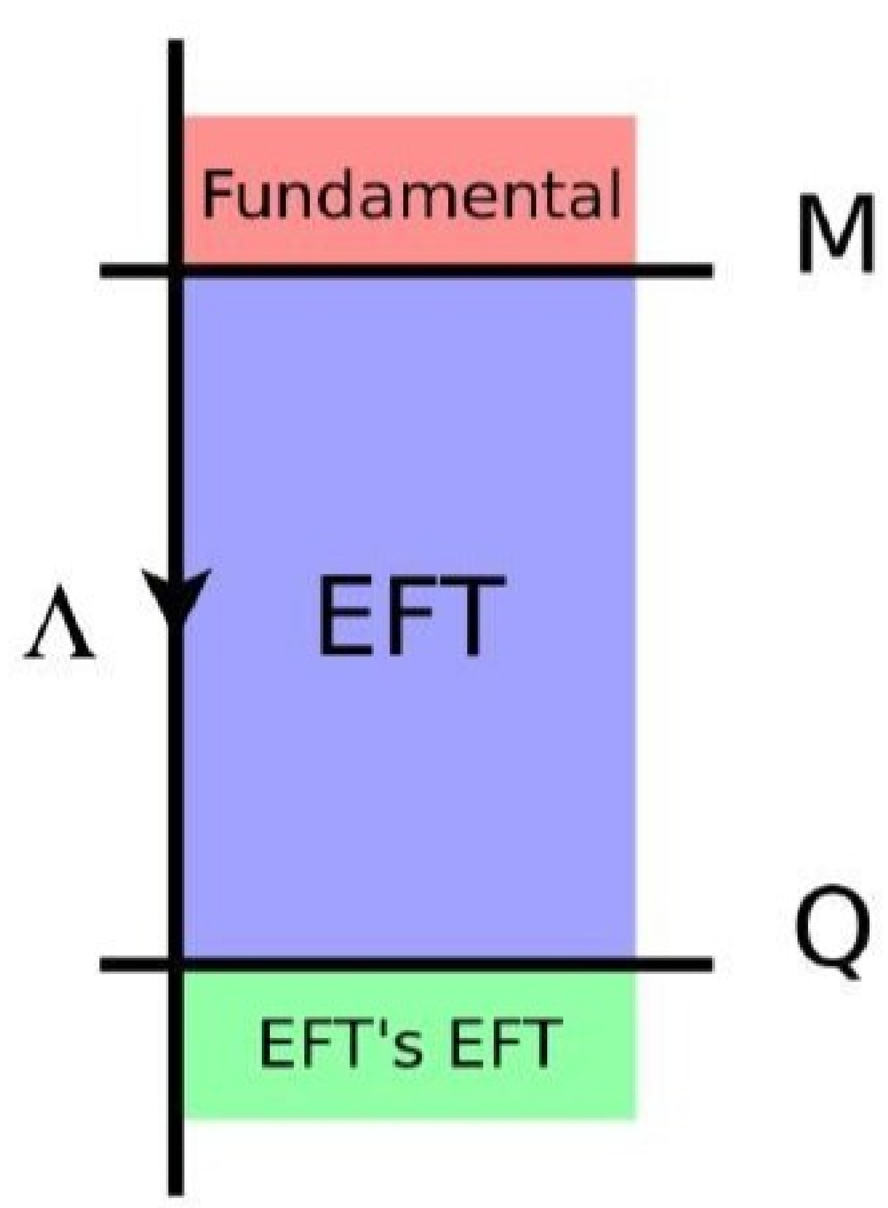
=D Have to $\underbrace{\text{grok}}_{}$ physics!

= have a deep understanding

We can do this w/ RENORMALIZATION

VIEW 1) $V_c(\vec{q}) = C_0(\Lambda)\, e^{-(\vec{q}/\Lambda)^2}$
as explanation for shallow bound states

VIEW 2) Let's try $\boxed{\text{RGA}}$



Physics is unique, but choice of theory depends on resolution Λ:

- $\Lambda \geq M$: Fundamental
- $M \geq \Lambda \geq Q$: EFT

For equivalent descriptions:

$$\boxed{\frac{d}{d\Lambda}\langle\Psi|\mathcal{O}|\Psi\rangle = 0}$$

Renormalization group invariance

But know we use it by its own, no reference to a concrete system

How to do this?

-> We begin w/ $\underbrace{\frac{d}{d\lambda}\langle\psi|\hat{O}|\psi\rangle = 0}$

But how does one interpret this?

-> First observation:

p- or r-space, 都可以

↳ let's choose r-space bc it's easier

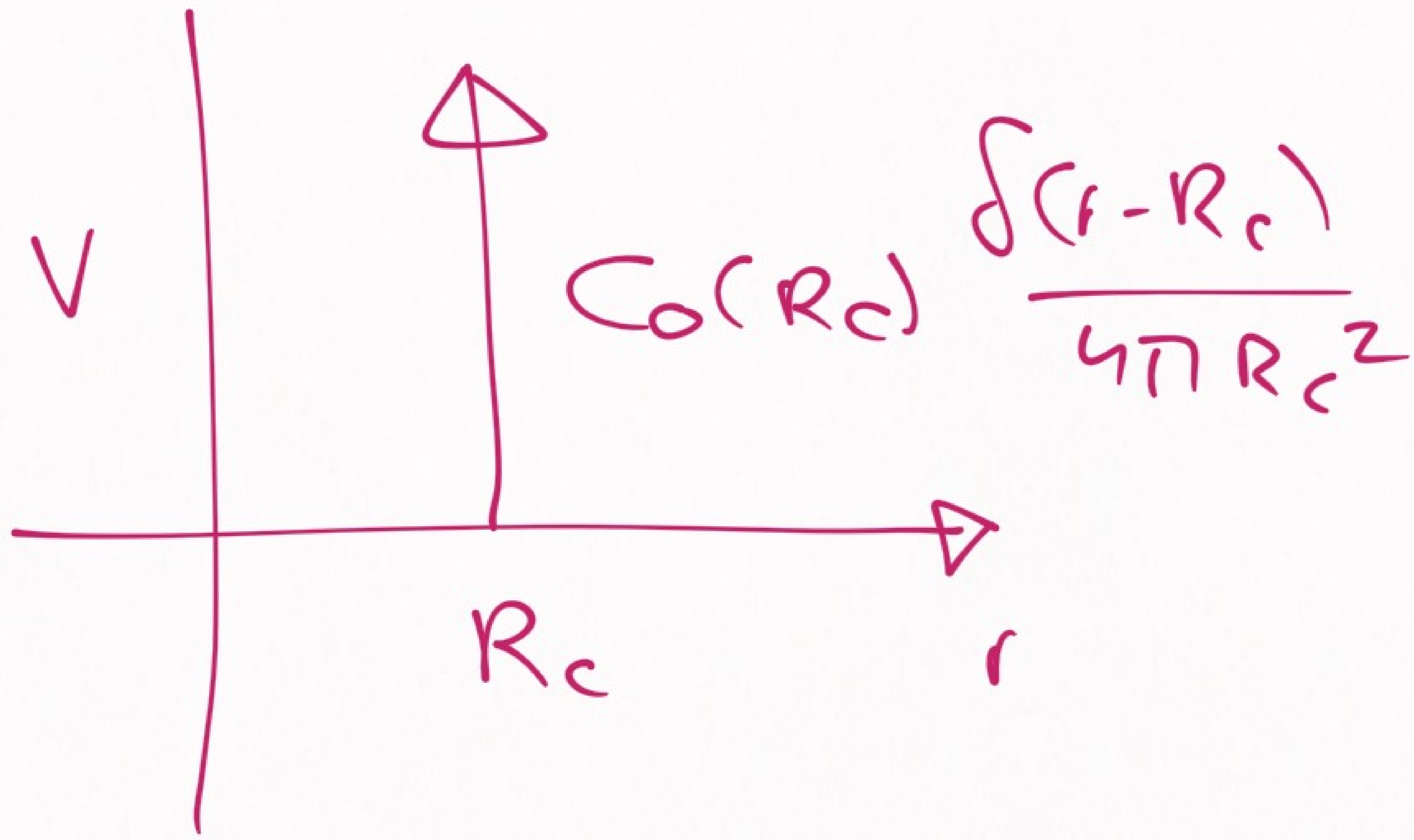
$$\Rightarrow \boxed{\frac{d}{dR_n}\langle\psi|\hat{O}|\psi\rangle = 0}$$

Next step → Dirac-delta in r-space

$$V_c(\vec{r}) = C_0 \, \delta^{(3)}(\vec{r})$$

$$\to V_c(r; R_c) = C_0(R_c) \, \frac{\delta(r - R_c)}{4\pi R_c^2}$$

this is called a delta-shell



Solution of the delta-shell:

$$\psi(\vec{r}) = \frac{1}{\sqrt{4\pi}} \frac{u(r)}{r}$$

$$\Rightarrow \lim_{\epsilon \to 0^+} \left[\frac{u'(R_c+\epsilon)}{u(R_c+\epsilon)} - \frac{u'(R_c-\epsilon)}{u(R_c-\epsilon} \right]$$

$$= \frac{2\mu C_0(R_c)}{4\pi R_c^2}$$

Reduced mass of the system

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$

— ⊗ —

$$\frac{d}{dR_c} \langle \psi | V_c | \psi \rangle = 0$$

Under control

But this?

Second ingredient: a wave function

Two possibilities come to mind:

1) $\psi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}$ (plane wave)

2) $\psi(\vec{r}) = \dfrac{A_s}{\sqrt{4\pi}}\dfrac{e^{-\gamma r}}{r}$ (bound state)

We try them:

1) $\langle\psi|V_c|\psi\rangle = \int d^3\vec{r}\, V_c(r;R_c)$

$= \int d^3\vec{r}\, C_0(R_c)\dfrac{\delta(r-R_c)}{4\pi R_c^2}$

$= C_0(R_c)$

$$\Rightarrow \boxed{\langle\psi|V_c|\psi\rangle = C_0(R_c)}$$

2) $\langle \psi | V_c | \psi \rangle =$

$$\int d^3\vec{r} \, \frac{|A_s|^2}{4\pi} \frac{e^{-2\gamma r}}{r^2} \, C(R_c) \, \frac{\delta(r - R_c)}{4\pi R_c^2} =$$

$$|A_s|^2 e^{-2\gamma R_c} \, \frac{C(R_c)}{4\pi R_c^2}$$

$$\boxed{\langle \psi | V_c | \psi \rangle \sim \frac{C(R_c)}{R_c^2}}$$

RG equations :

1) $\frac{d}{dR_c}\left[C(R_c)\right] \simeq 0$

2) $\frac{d}{dR_c}\left[\frac{C(R_c)}{R_c^2}\right] \simeq 0$

They are not the same!

So what is going on here?

Remember the teacup & teapot effective theory?

"CUP" POWER COUNTING

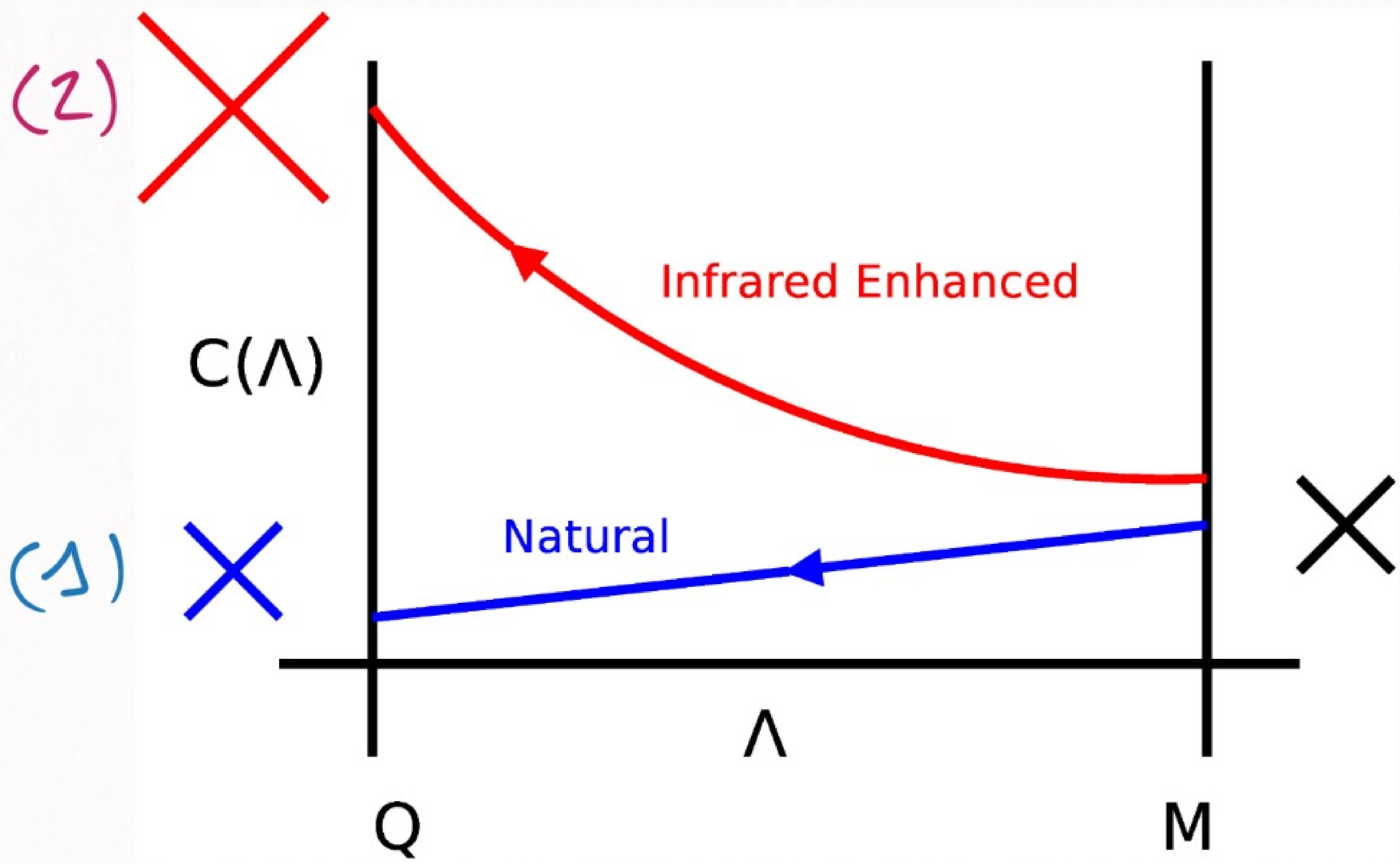
"POT" POWER COUNTING

→ The same is happening w/ RGE (1) & (2)

(1) $\frac{d}{dR_c}\left[C(R_c)\right]=0$

(2) $\frac{d}{dR_c}\left[\frac{C(R_c)}{R_c^2}\right]=0$

First take-home message:



RGE have multiple solutions

(1) $\frac{d}{dR_c} C(R_c) \simeq 0 \rightarrow$ natural system

(2) $\frac{d}{dR_c}\left[\frac{C(R_c)}{R_c^2}\right] \simeq 0 \rightarrow$ unnatural system

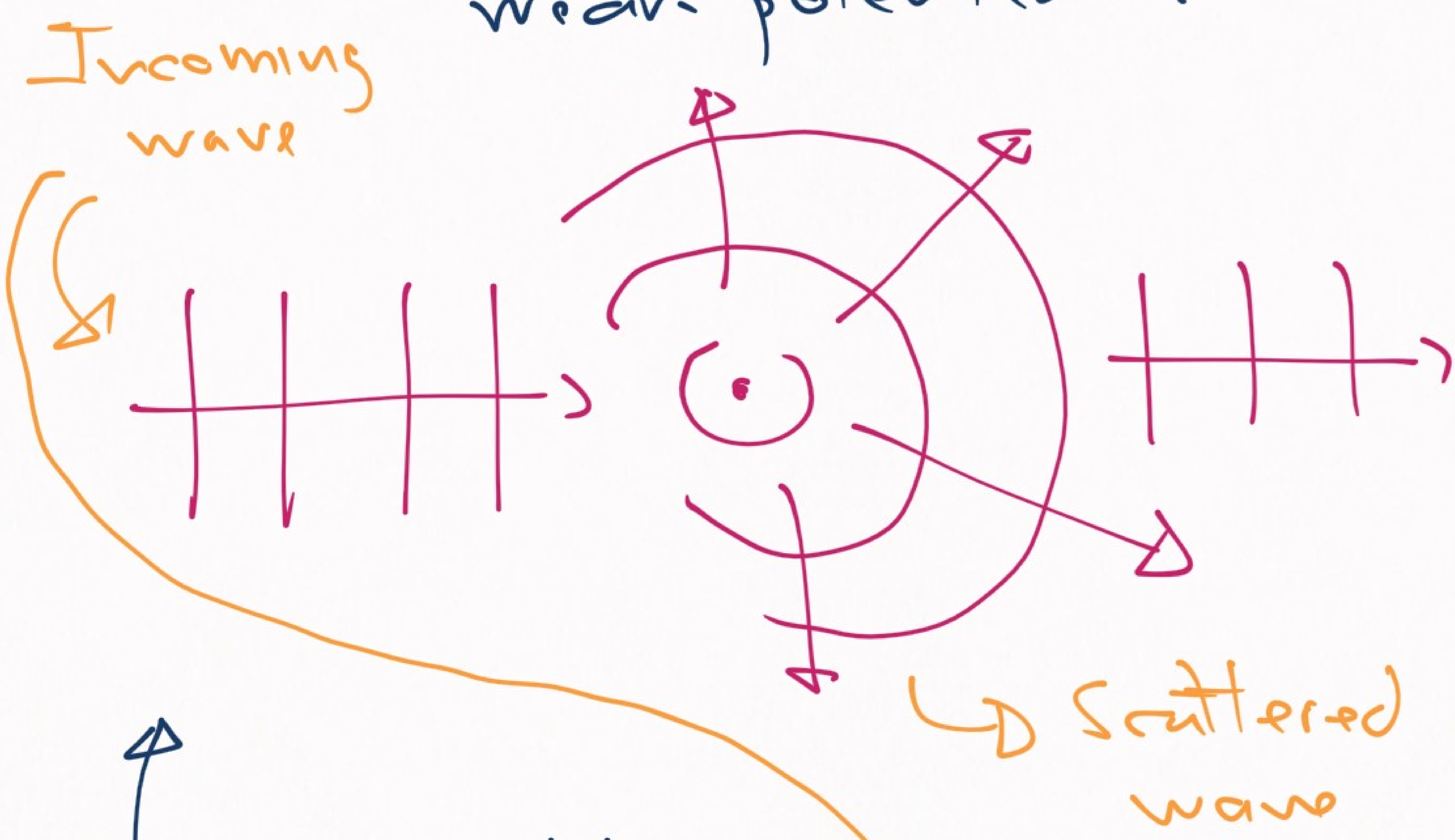
aka "infrared enhanced" Why?

(IR limit) $\lim_{R_c \to \infty} C(R_c) \to \infty$

Let's study which type of system do solutions 1) & 2) describe

1) Key assumption: $\psi(\vec{r}) \simeq e^{i\vec{k}\cdot\vec{R}}$

→ Scattering by a weak potential:



Typical scattering setup

$$\psi(\vec{r}) \xrightarrow[r\to\infty]{} e^{i\vec{k}\cdot\vec{r}} + f(\hat{r})\frac{e^{ikr}}{r}$$

Scattering wave function:

$$\psi(\vec{r}) \longrightarrow e^{i\vec{k}_i\cdot\vec{r}} + f(\hat{r})\frac{e^{ikr}}{r}$$

Weak potential $\rightarrow$ Born approximation

$$f(\hat{r}) \simeq -\frac{\mu}{2\pi}\int d^3\vec{r}\, V(\vec{r})\, e^{-i(\vec{k}_f-\vec{k}_i)\cdot\vec{r}}$$

↳ the point is that:

$$\left|e^{i\vec{k}_i\cdot\vec{r}}\right| \gg \left|f(\hat{r})\frac{e^{ikr}}{r}\right|$$

$\Rightarrow$ Justifies our choice of a wave function in

$$\boxed{\frac{d}{dR_c}\langle\psi|V_c|\psi\rangle = 0}$$

Example: scattering by a weak (perturbative) Yukawa potential

$$V(\vec{r}) = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}$$

$$\Rightarrow f_{\underline{k}}(\hat{r}) = \frac{\mu}{2\pi}\frac{g^2}{(\vec{k}-\vec{k}')^2+m^2}$$

$$\simeq \frac{\mu}{2\pi}\frac{g^2}{m^2}\left[1 - \frac{(\vec{k}-\vec{k}')^2}{m^2} + \dots\right]$$

At low energies we have:

$$\boxed{f_{\underline{k}}(\hat{r}) \simeq \frac{\mu}{2\pi}\frac{g^2}{m^2}}$$

1) How does EFT describes this?

2) What is the running of $C_0(R_c)$?

1) Description:

$$V_c(\vec{r}\,; R_c) = C_0(R_c)\,\frac{\delta(r-R_c)}{4\pi R_c^2}$$

$$\Rightarrow f_{EFT}(\hat{n}) = -\frac{\mu\, C_0(R_c)}{2\pi}\,\frac{\sin(q R_c)}{(q R_c)}$$

w/ $q = |\vec{k} - \vec{k}'|$

$$\Rightarrow \left[C_0(R_c) = -\frac{g^2}{m^2} \right]$$

(matches $f_Y(\hat{r})$ modulo $\mathcal{O}(q^2)$ terms)

2) Running: $\boxed{\frac{d}{dR_c} C_0(R_c) = 0}$

→ reproduces our RGE

Take-home message:

The RGE $\frac{d}{dR_c} C(R_c) = 0$

describes a "natural system"

(one for which the Born approximation works well)

Besides, do not forget solution 2):

The RGE $\frac{d}{dR_c}\left[\frac{C(R_c)}{R_c^2}\right] = 0$

describes a "fine-tuned system"

(one for which there is a shallow bound state)

↳ Exercise: What about a bound state that is deep? (2 points)

By the way, this is also the core idea in the manuscript

3. **A Renormalization group treatment of two-body scattering**
(196) Michael C. Birse, Judith A. McGovern, Keith G. Richardson (Manchester U.). Jul 1998. 4 pp.
Published in **Phys.Lett. B464 (1999) 169-176**
MC-TH-98-11
DOI: 10.1016/S0370-2693(99)00991-0
e-Print: **hep-ph/9807302** | **PDF**

References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service

except that they use p-space

(could be a good extra reading)

— ⊗ —

A possible exercise:

→ Repeat the previous RGA w/ a Gaussian regulator

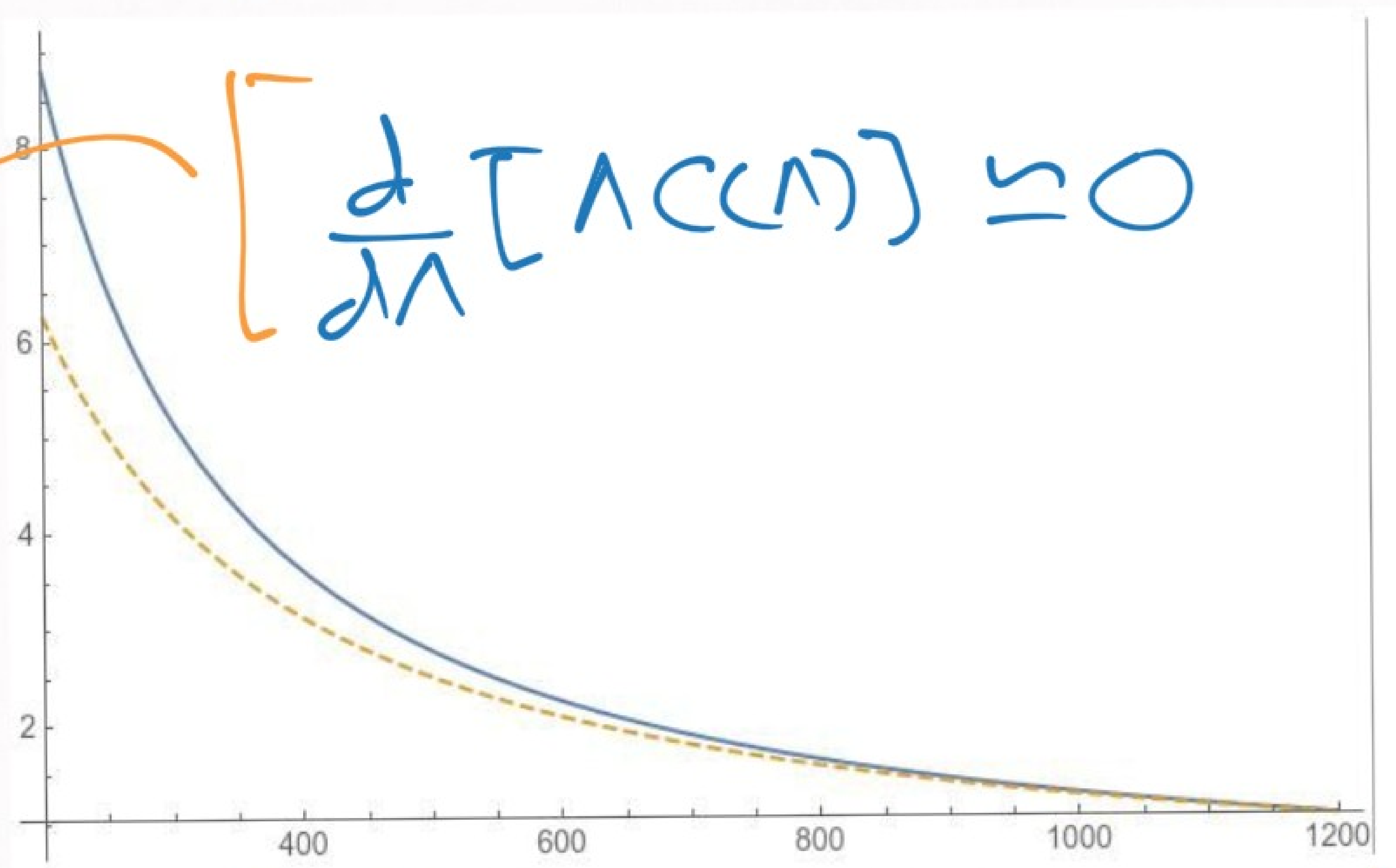
$$V_c(r;R_c) = C_0(R_c)\,\frac{e^{-(r/R_c)^2}}{\pi^{3/2} R_c^3}$$

(1 pt, or 2 pts if you do it without explicit evaluation of the integrals)

(style)

But here there's a really interesting conceptual problem...

Previously we found that:



$$\frac{d}{d\Lambda}[\Lambda C(\Lambda)] \simeq 0$$

which in r-space reads:

$$\boxed{\frac{d}{dR_c}\left[\frac{C(R_c)}{R_c}\right] \simeq 0}$$

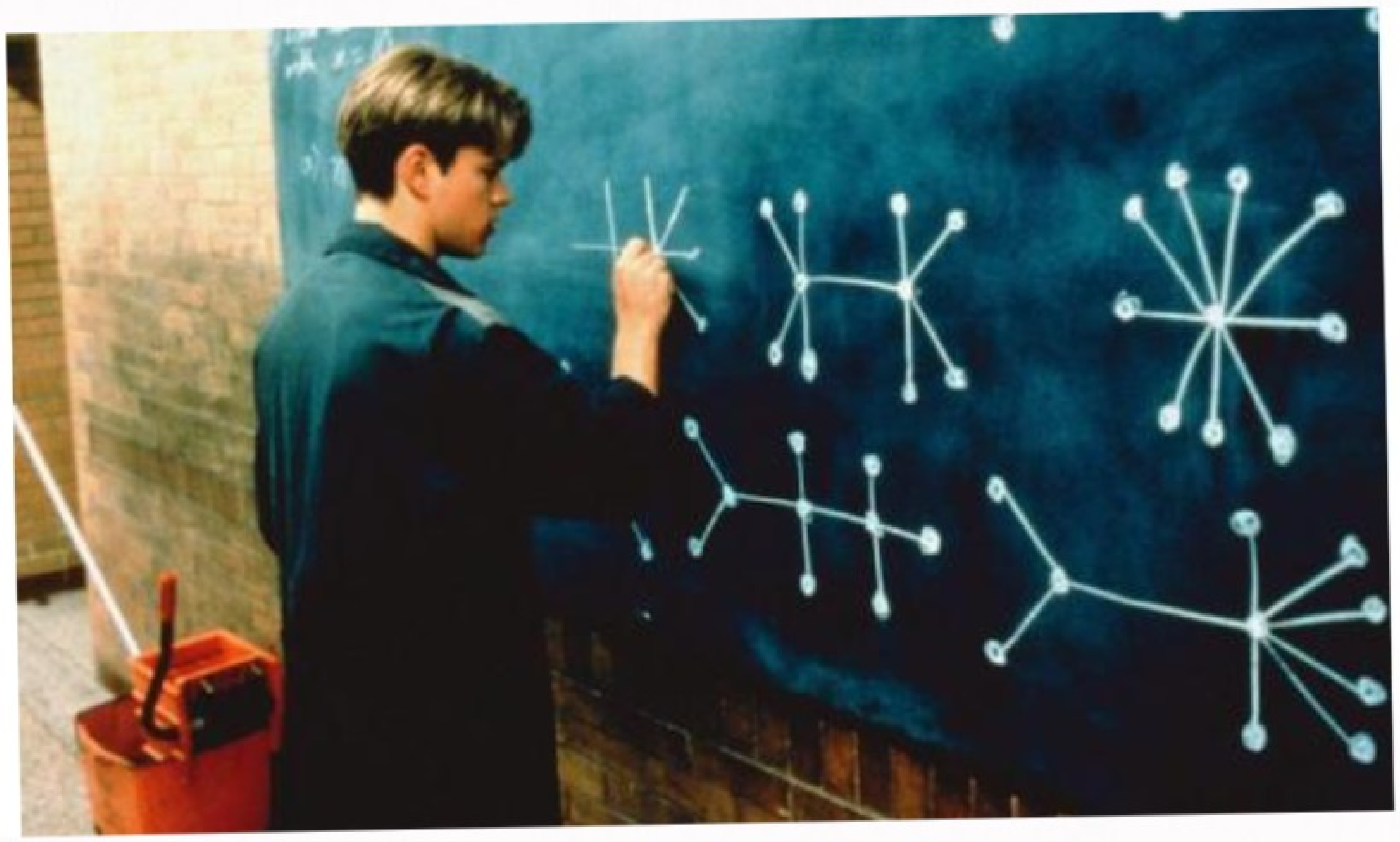
What!?

But before we got...

$$\boxed{\frac{d}{dR_c}\left[\frac{C(R_c)}{R_c^2}\right] \simeq 0}$$

So what is going on here?

→ This is a damn good problem!



(your "Good Will Hunting" moment)

→ Most EFT theorists I know don't know the answer, so...

1) 8 points to the first person that fully solves it
2) 5 points to the second
3) 3 points to the rest

(it's a conceptual problem)

[More RGA & Feynman 六之道]

We have derived RGE & power counting from:

1) Fitting $V_c = C_0(\Lambda)\, e^{-(\vec{q}/\Lambda)^2}$ to a bound state

2) Solving $\frac{d}{d\Lambda}\langle\psi|\hat{O}|\psi\rangle = 0$ directly

But there more ways:

3) Directly counting powers of Q and M:

3. **Effective field theory of short range forces**
U. van Kolck (Caltech, Kellogg Lab & Washington U., Seattle). Aug 1998. 38 pp.
Published in **Nucl.Phys. A645 (1999) 273-302**
KRL-MAP-230, NT-UW-98-01
DOI: 10.1016/S0375-9474(98)00612-5
e-Print: **nucl-th/9808007** | **PDF**

4) Directly removing the infinities:

7. **Perturbative renormalizability of chiral two pion exchange in nucleon-nucleon scattering**
(90) M.Pavon Valderrama (Julich, Forschungszentrum & JCHP, Julich). Dec 2009. 4 pp.
Published in **Phys.Rev. C83 (2011) 024003**
FZJ-IKP-TH-2009-37
DOI: 10.1103/PhysRevC.83.024003
e-Print: **arXiv:0912.0699** **[nucl-th]** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service; Link to Fulltext
Detailed record - Cited by 90 records 50+

5) Looking at the "residual cutoff dependence"

1. **Renormalizing Chiral Nuclear Forces: Triplet Channels**
(68) Bingwei Long (Jefferson Lab), C.J. Yang (Arizona U. & Ohio U., Inst. Nucl. Part. Phys.). Nov 2011. 20 pp.
Published in **Phys.Rev. C85 (2012) 034002**
JLAB-THY-11-1464, INT-PUB-11-038
DOI: 10.1103/PhysRevC.85.034002
e-Print: **arXiv:1111.3993** **[nucl-th]** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service; OSTI.gov Server; JLab Document Server
Detailed record - Cited by 68 records 50+

6) Considering matrix elements of some fundamental potential w/ effective wave functions

$$\langle \psi_L | V_S | \psi_L \rangle \longrightarrow \left[\begin{array}{l} V_S \to \text{"true" } V \\ \psi_L \to \text{effective } \psi \end{array} \right.$$

$$V_s(r) = M f(Mr)$$

$$\psi_L(r) = (Qr)^n \rightarrow$$ only interested in short-range

$\rightarrow$ make it dimensionless

$$\langle \psi_L | V_s | \psi_L \rangle \simeq \left(\frac{Q}{M}\right)^{2n} I_s(2n)$$

$$\text{w/ } I_s(k) = \int_0^\infty dx\, f(x)\, x^k$$

1) Natural case: $\psi_L \sim e^{i\vec{k}\cdot\vec{r}}$

$$\sim e^{iQr}$$

$$\sim 1$$

$\Downarrow$

$$\langle \psi_L | V_s | \psi_L \rangle \simeq \left(\frac{Q}{M}\right)^0 I_s$$

2) Unnatural case: $\psi_L \sim \frac{e^{-\gamma r}}{r}$

$$\sim \frac{e^{-Qr}}{Qr}$$

$$\langle \psi_L | V_s | \psi_L \rangle \simeq \underbrace{\left(\frac{M}{Q}\right)^2}_{\text{IR ENHANCEMENT}} I_s'$$

$$\sim \frac{1}{Qr}$$

But most of you probably will never need this level of mastery

So....

NEXT CLASS:

[BACK TO NORMAL]