Nuclear Physics 8

Effective Field Theories

RECAP:

→ Renormalization:

Physics at long-distances does not depend on short-range details

→ RGA:

$$\frac{d}{d\Lambda}\langle\Psi|O|\Psi\rangle = 0$$

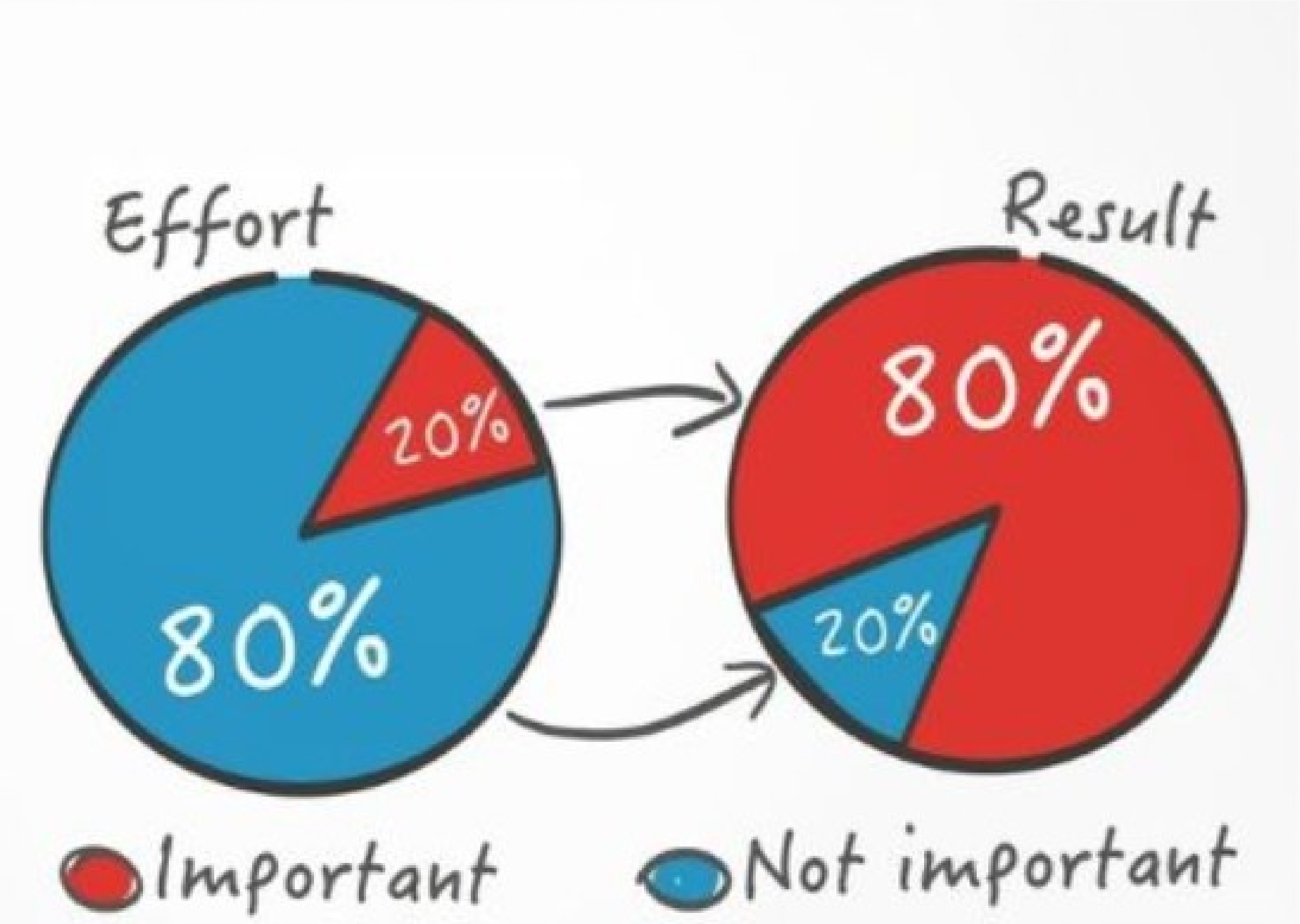
$\Downarrow$

Foundation of EFTs

EXAMPLES OF EFT "思维"
(EFT "way of thinking")

1) Teacup & Teapot theory

2) Pareto Principle

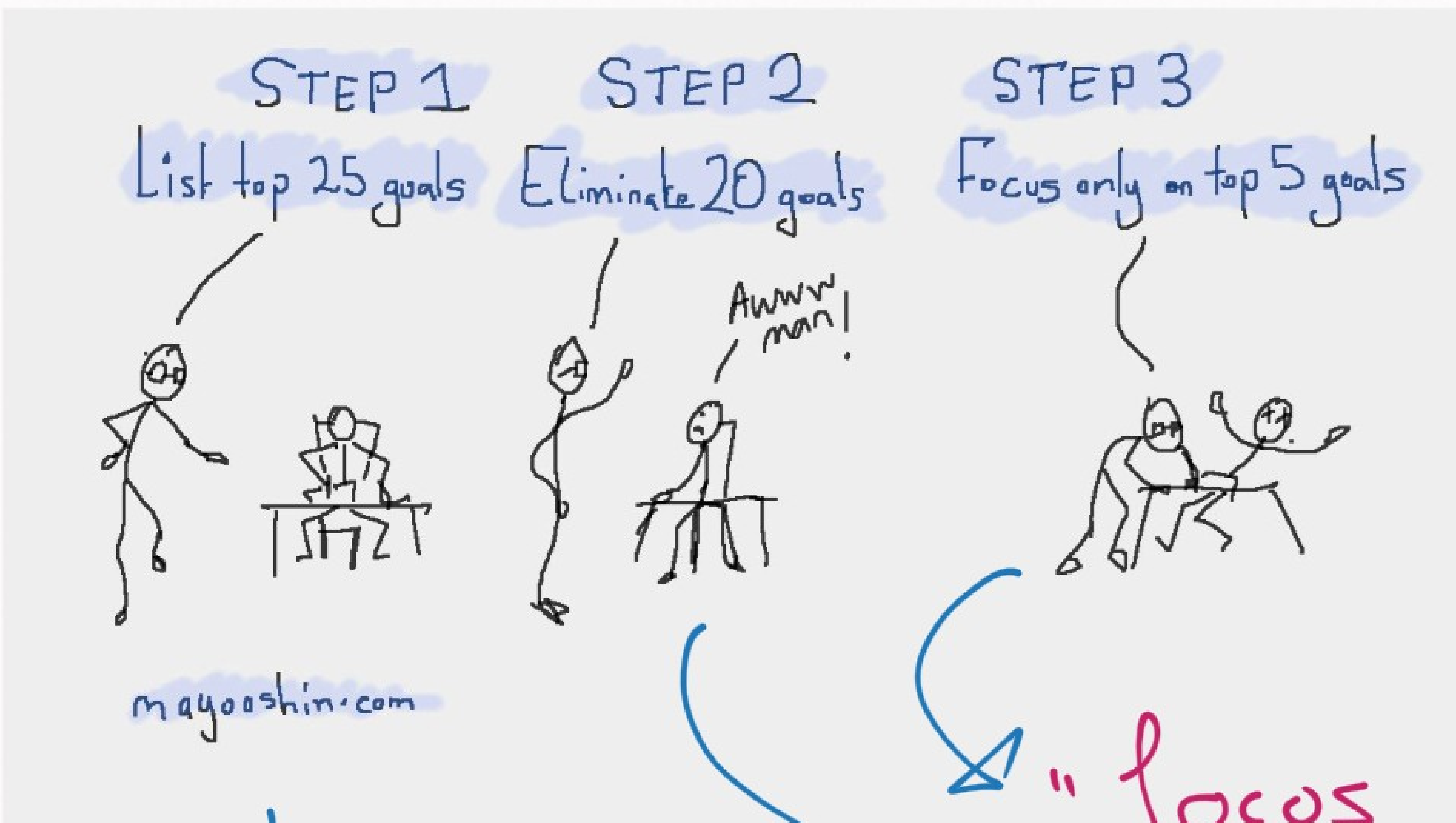


"20% of effort gives 80% of results"

→ clear example of POWER COUNTING

3) Warren Buffet's 5/25 rule

(EFT 思维 might make you rich)

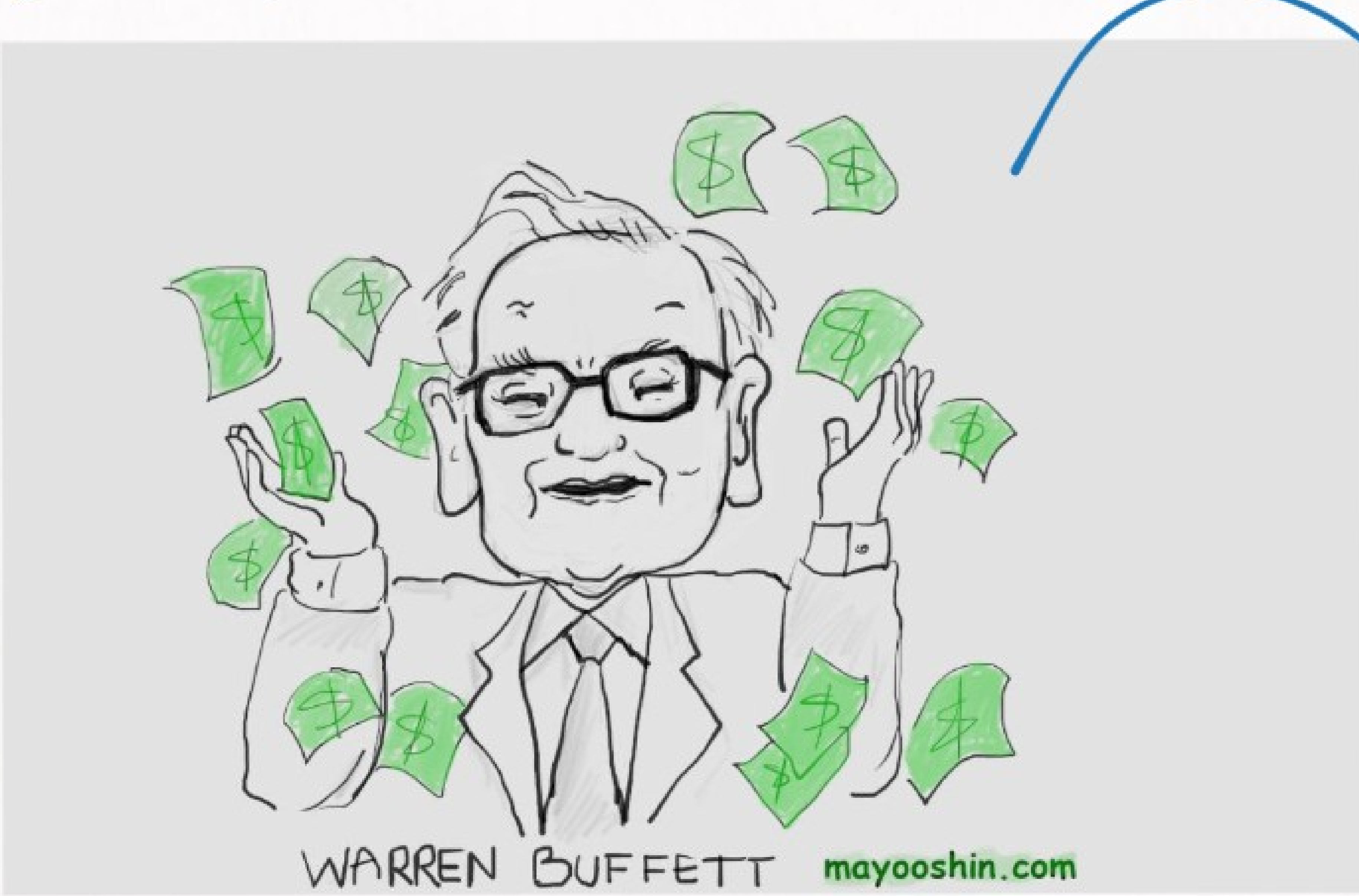


"focos on LO"

"write down all possible interactions"

"set up a power counting"

结果/Outcome:



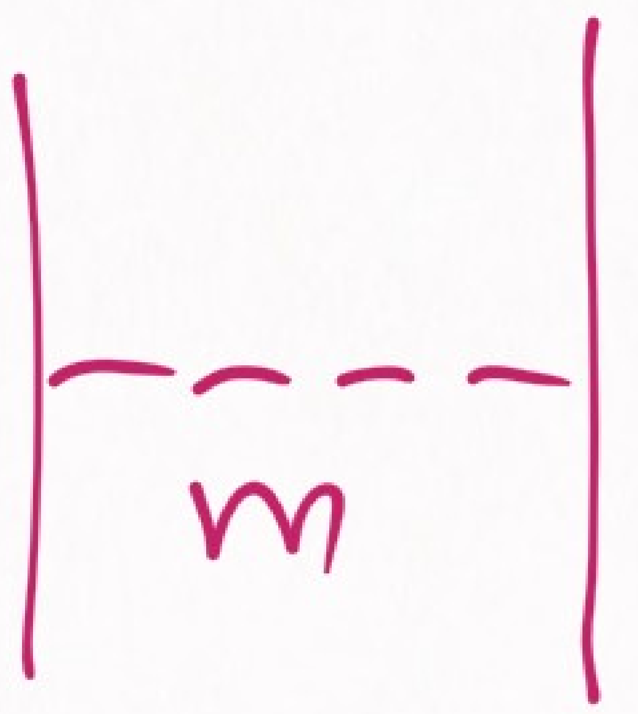
"get those sweet citations"

THE POINT $\Rightarrow$ IT'S A POWERFUL WAY OF THINKING

4) Even in Art: Impresionism
(印象派)

→ Big dots
(1→Q)
→ No short distance detail
→ Still a great painting
(an "effective" painting)

Let's go back to PHYSICS:



$$\longrightarrow \quad V(r) = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}$$

$$V(\vec{q}) = -\frac{g^2}{m^2+|\vec{q}|^2}$$

Two-body problem:

1) two particles of mass M
2) they exchange a scalar boson of mass m
3) the potential is the one written above

$$\Rightarrow \left[-\frac{\nabla^2}{M}+V(r)\right]\psi(\vec{r}) = E\,\psi(\vec{r})$$

This two-body system binds if:

$$\lambda = \frac{M}{m}\frac{g^2}{4\pi} \geq \underbrace{1.68}_{=\lambda_c}$$

1) $\lambda < \lambda_c \Rightarrow$ no binding

2) $\lambda = \lambda_c \Rightarrow$ zero energy bound state

(aka: unitary limit)

3) $\lambda > \lambda_c \Rightarrow$ binding

We will take:

$M = 1$ GeV

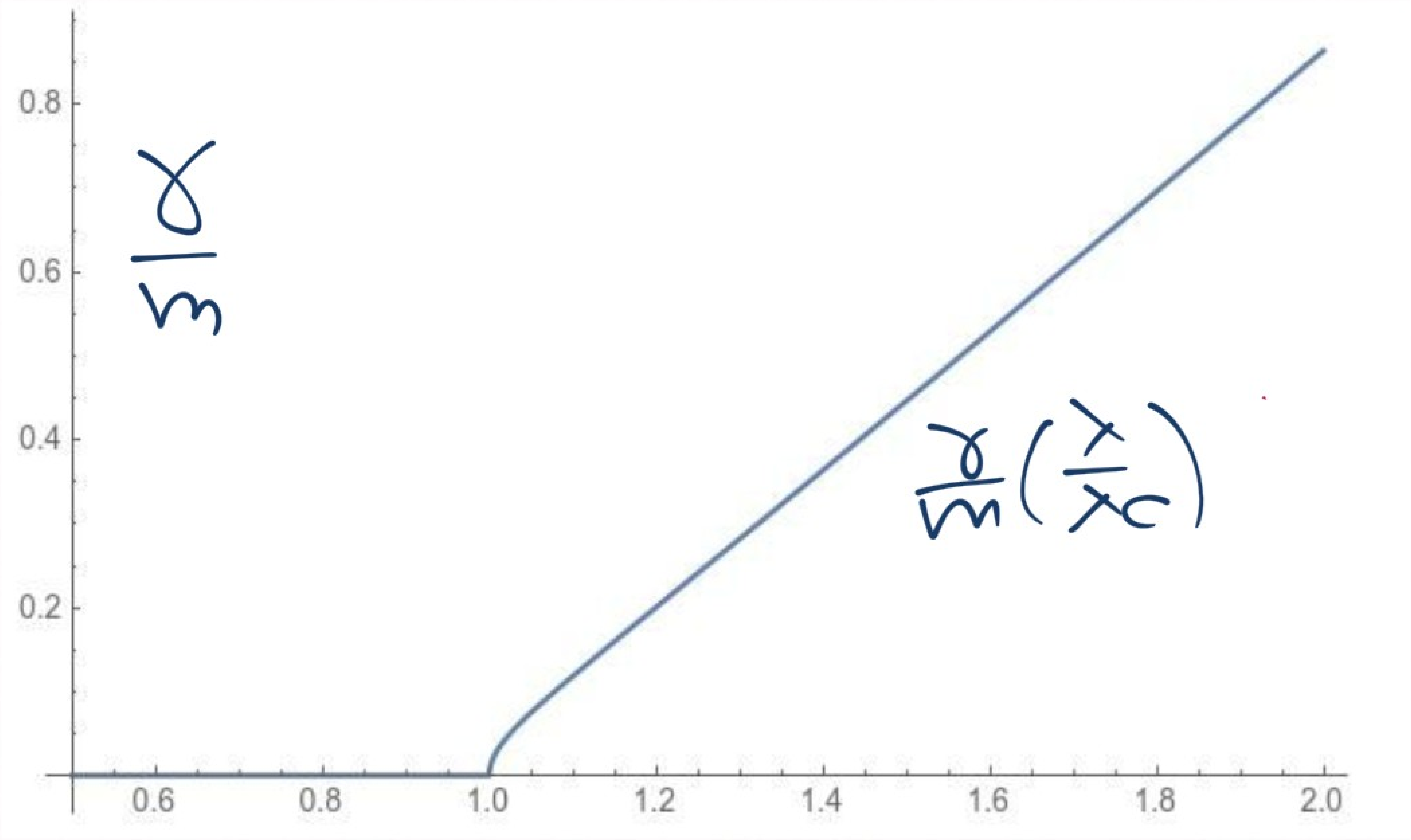
$m = 0.5$ GeV

$E_B = -\frac{\gamma^2}{M} \Rightarrow$ We can compute

$$[\gamma = \gamma(\lambda)]$$

Even better, we can write γ in units of m and λ_c:

$$\left[\frac{\gamma}{m} = \frac{\gamma}{m}\left(\frac{\lambda}{\lambda_c}\right)\right]$$



λ/λ_c

no binding $\longleftarrow$ $\longrightarrow$ binding

For $\frac{\lambda}{\lambda_c} \to 1$, $\frac{\gamma}{m} \to 0$

$\Rightarrow$ The spatial extend of the wave function will be much larger than the range of the potential

$\Downarrow$

WHY? $\psi_s(\vec{r}) = \frac{1}{\sqrt{4\pi}} \frac{u(r)}{r}$

w/ $u(r) \longrightarrow A_s e^{-\gamma r}$

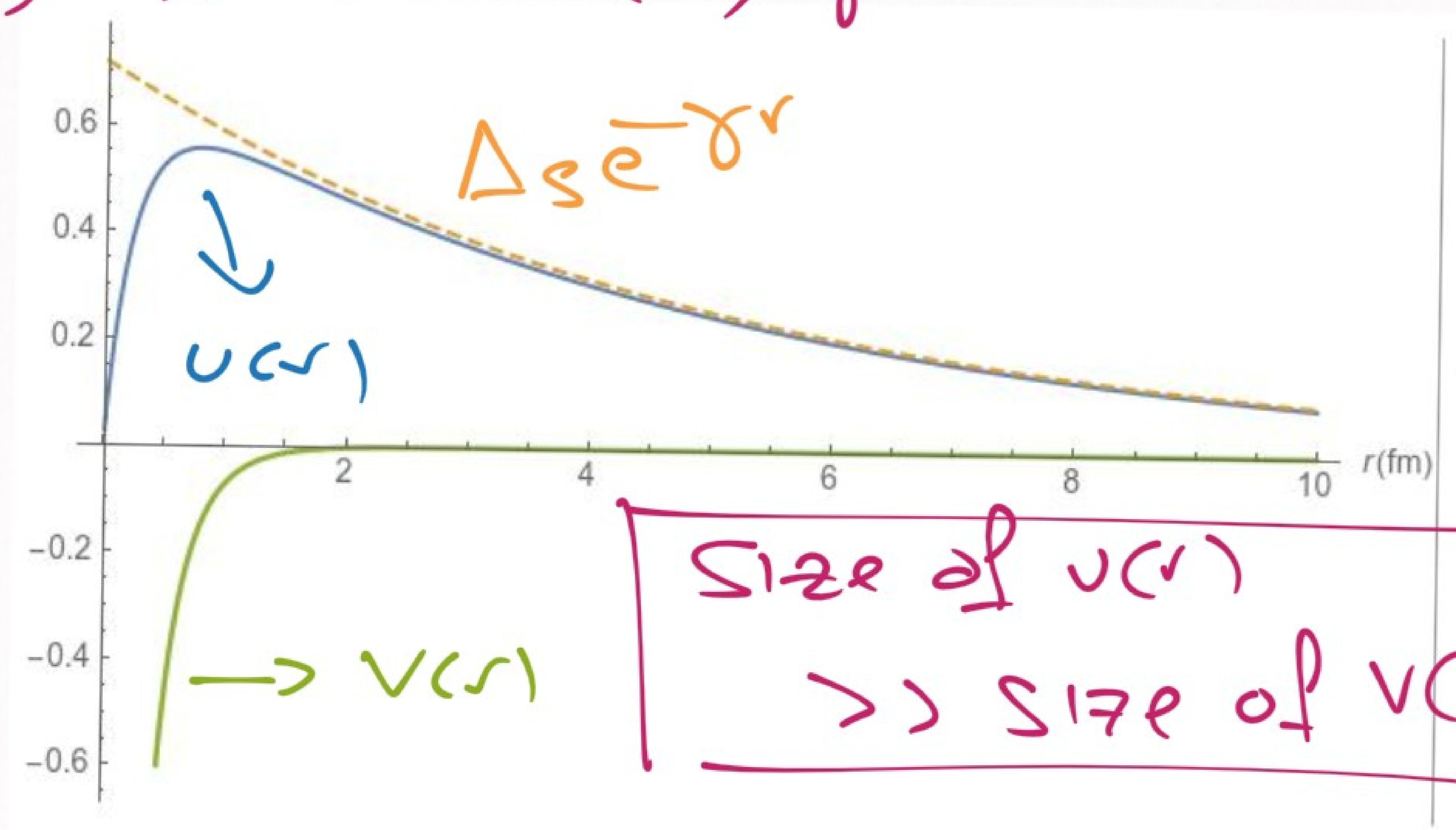
for $r \gg \frac{1}{m}$

$\Rightarrow$ the size of the wavefunction is given by $1/\gamma$

$\Downarrow$

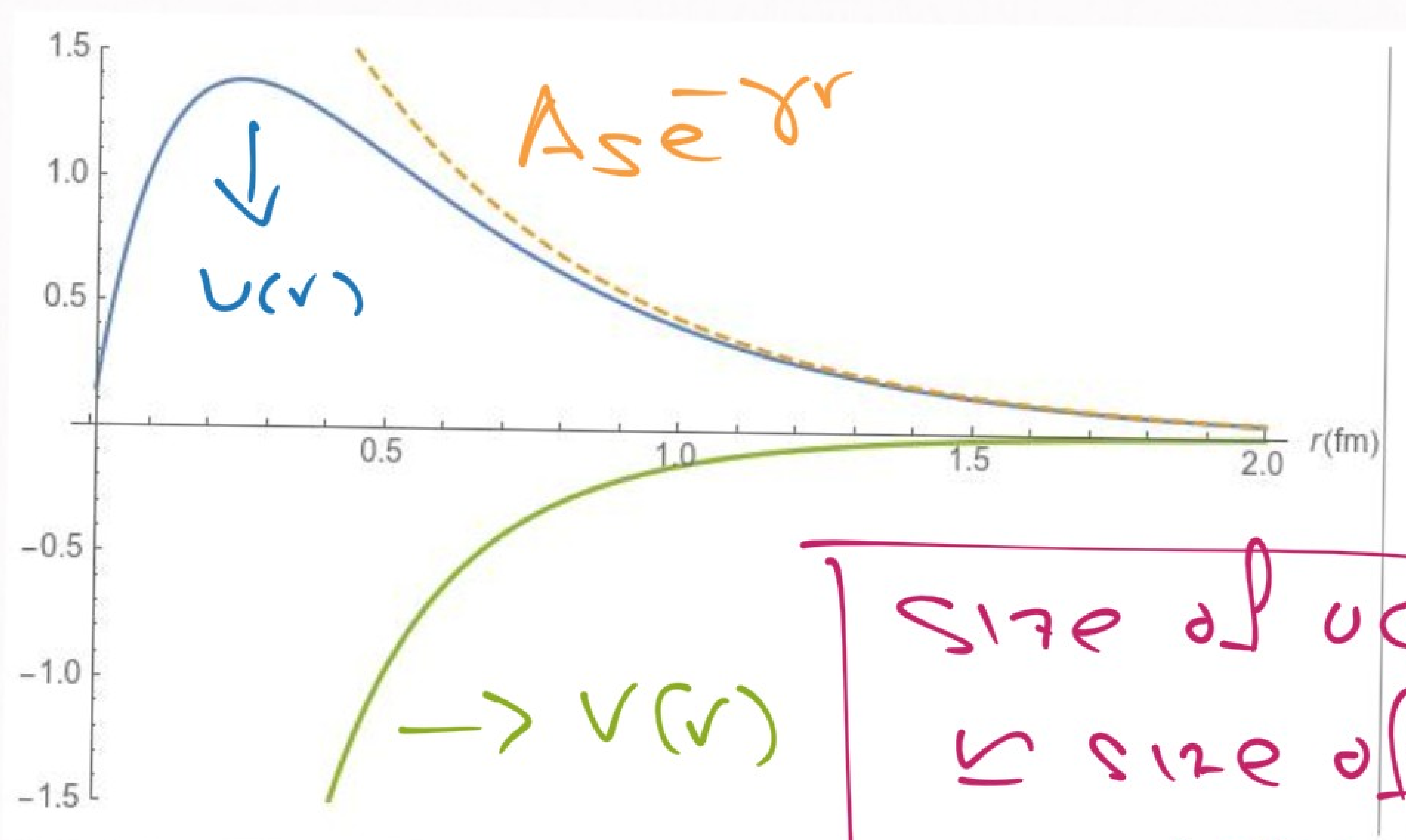
We can check this w/ examples:

a) $\lambda = 1.05\lambda_c$, $\gamma = 0.075m$



$\boxed{\text{Size of } u(r) \gg \text{size of } V(r)}$

b) $\lambda = 2\lambda_c$, $\gamma = 0.86m$



$\boxed{\text{Size of } u(r) \lesssim \text{size of } V(r)}$

What does this mean?

a) Unnatural

b) Natural

But also:

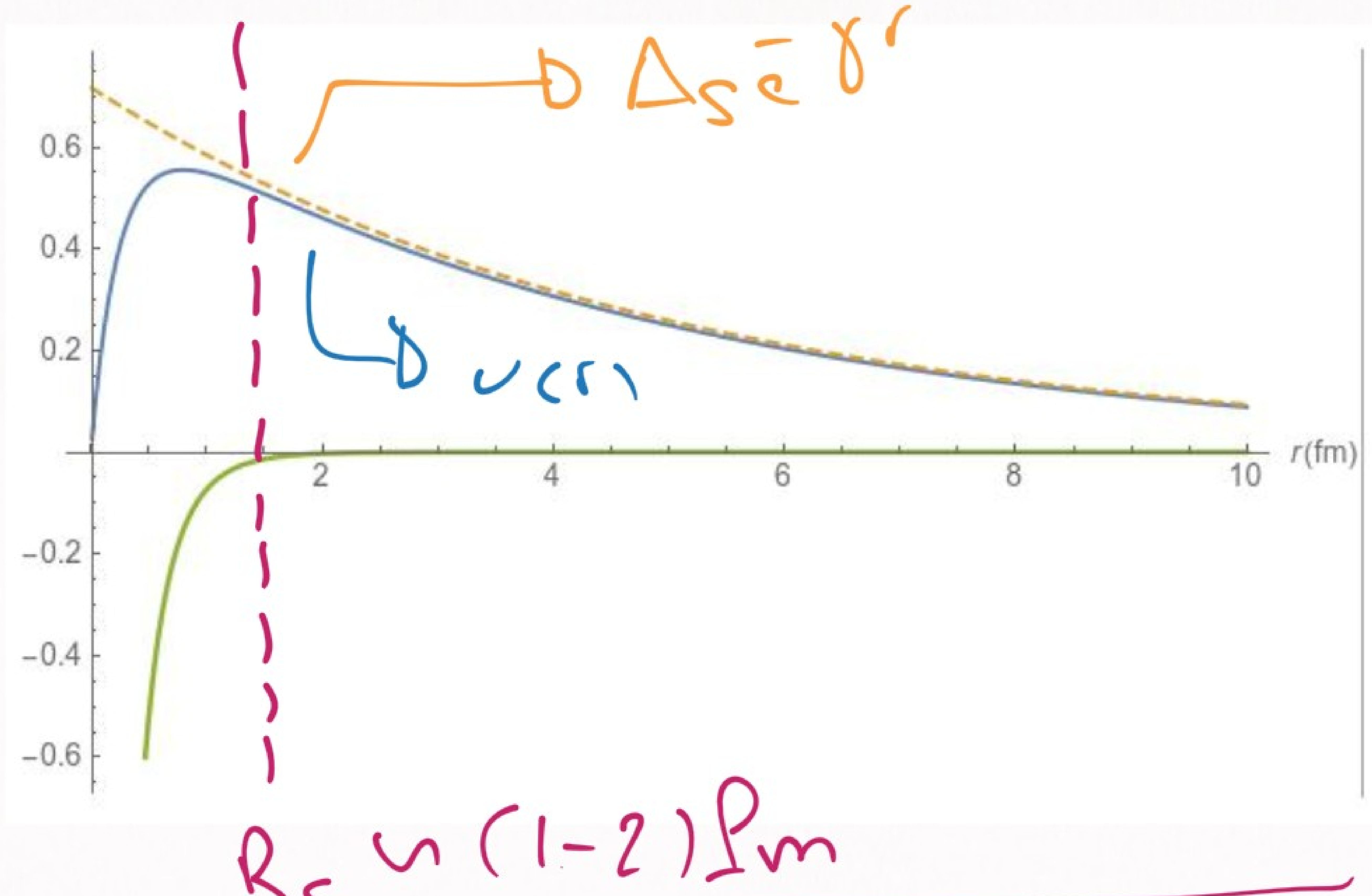
a) Wave function insensitive to the form of $V(r)$

b) Wave function sensitive to the form of $V(r)$

That is:

$\Rightarrow$ For $\frac{\lambda}{\lambda_c} \to 1$, it is possible to build an effective description of this system

Let's take a second look:



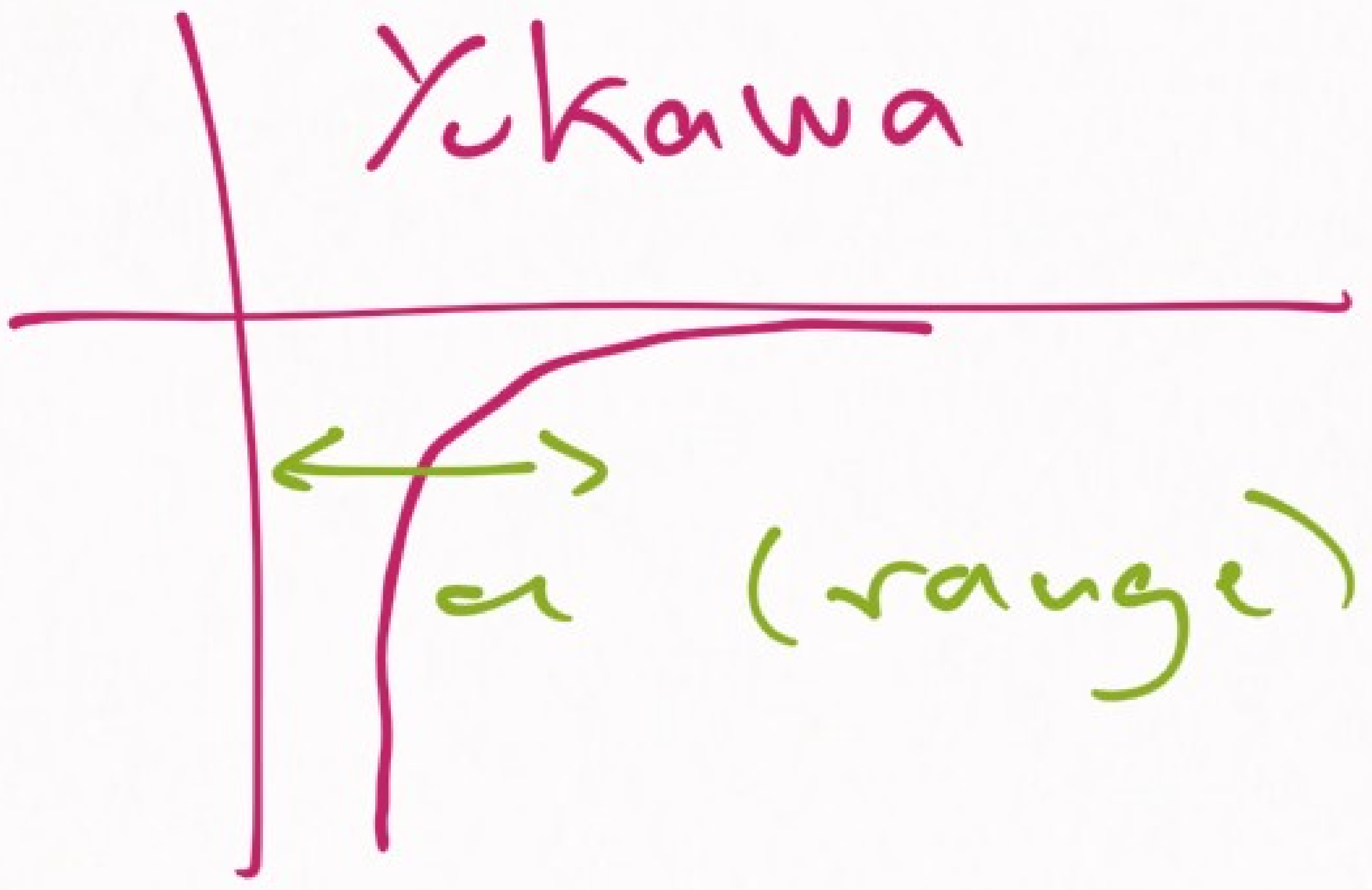
$$\boxed{\text{For } r > R_c,\ u(r) \simeq A_s e^{-\gamma r}}$$

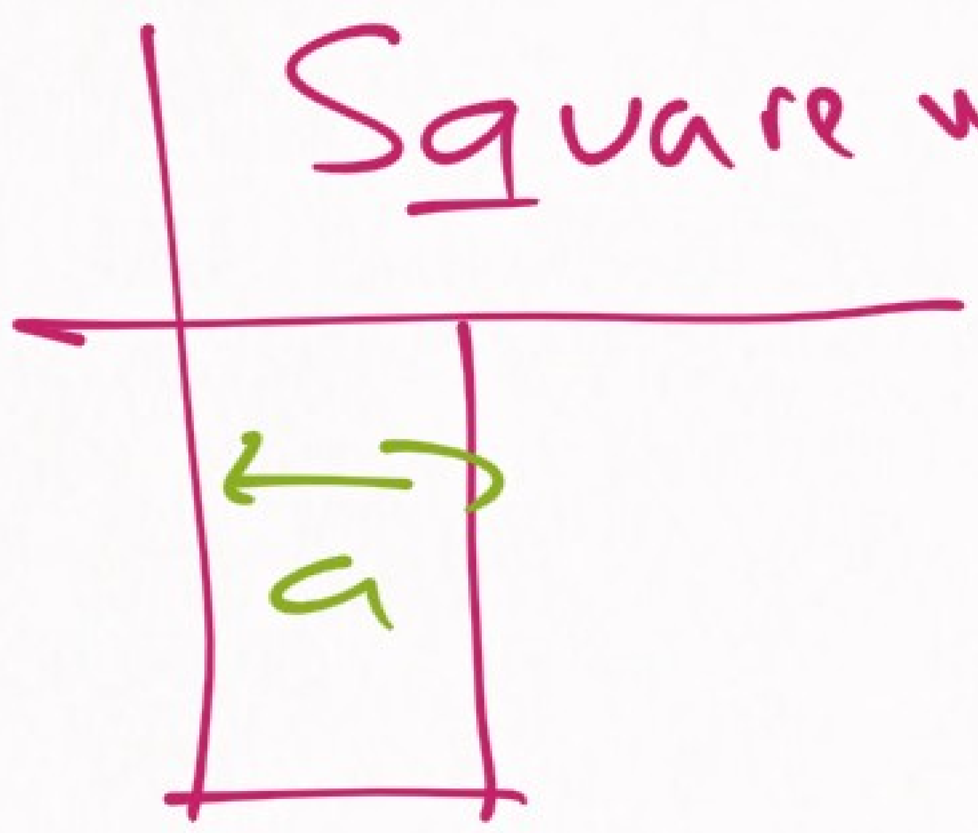
$\to$ they are indistinguishable

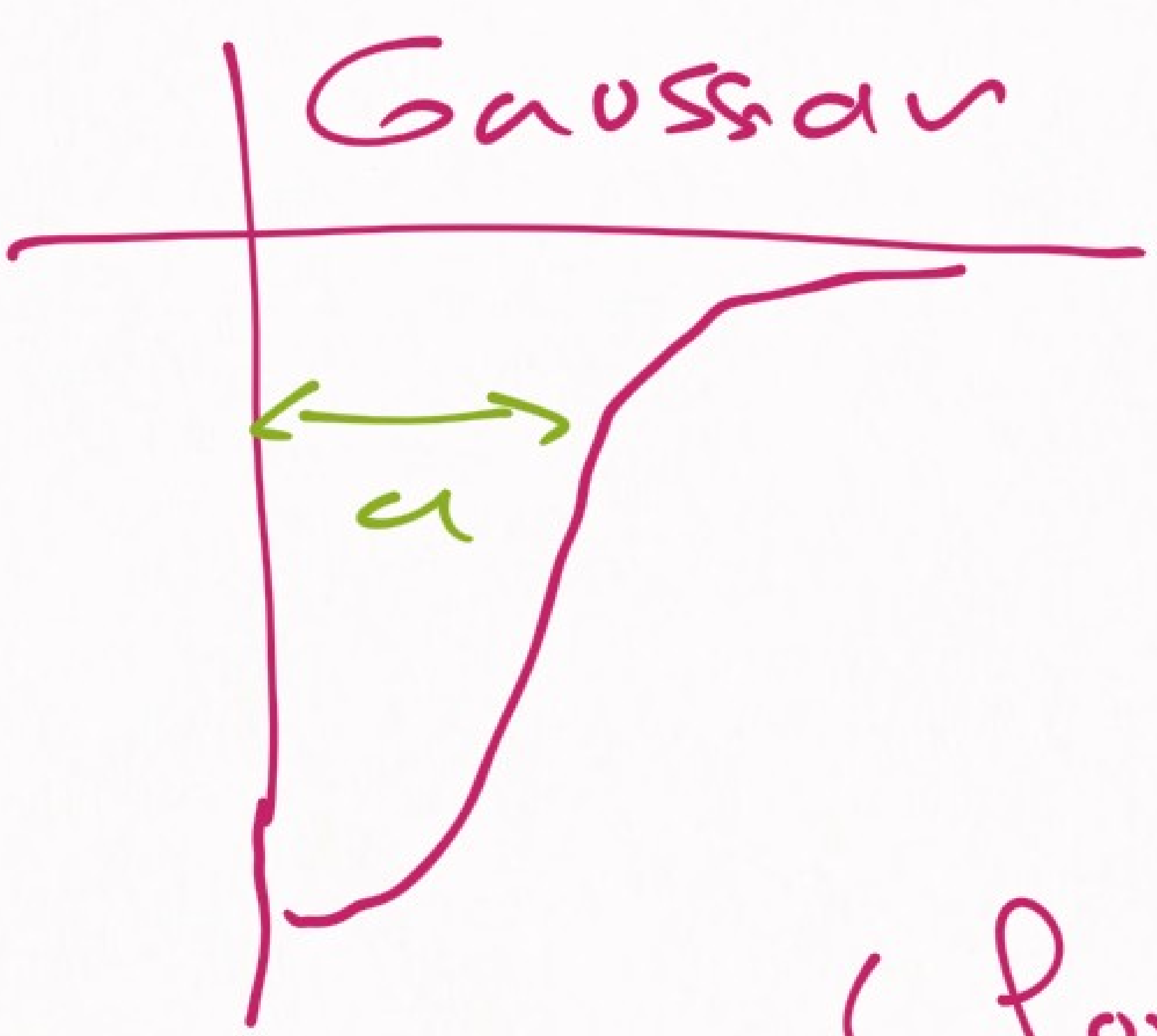
Besides, most of the wavefunction lives at $r > R_c$

$\to$ the piece inside $r < R_c$ is negligible

This means that for the a) system it doesn't matter what is the potential $V(r)$







All of them will give you

$$u(r) \simeq A_s e^{-\gamma r}$$

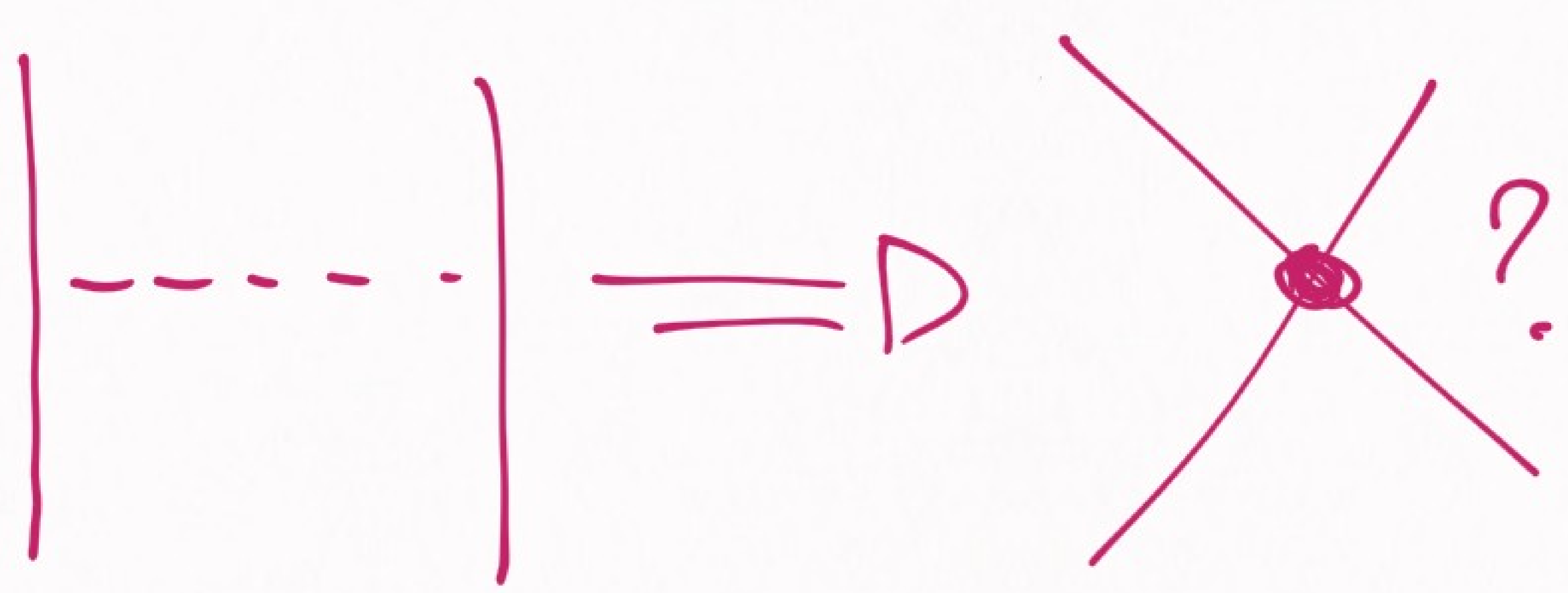
$\left(\text{for } \frac{1}{\gamma} >> a \right)$

Remember: PHYSICS AT LONG DISTANCES DOES NOT DEPEND ON SHORT DISTANCE DETAILS

So if $\frac{1}{\gamma} >> a$, we don't need $\underbrace{\text{the exact potential}}_{\text{it does not matter}}$

$\Downarrow$

We can substitute it by something more simple

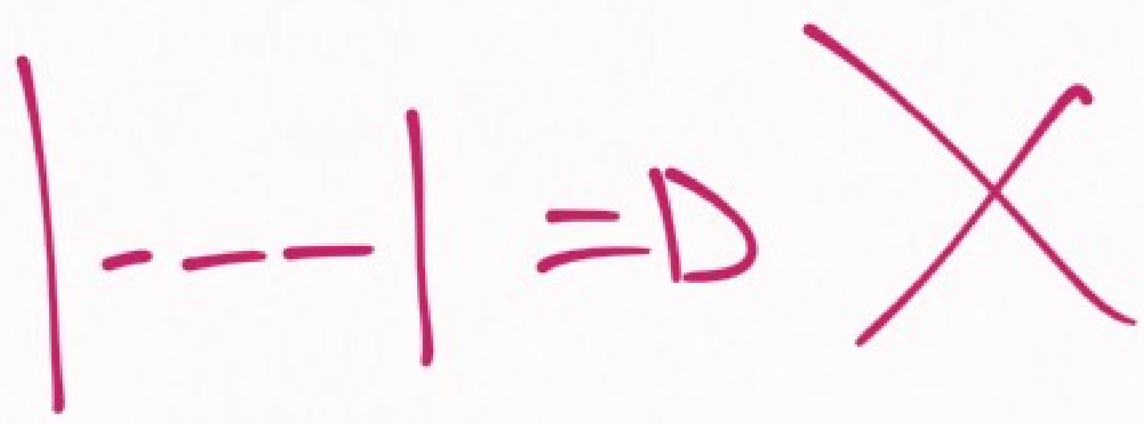


$$V(\vec{p}-\vec{p}') = \frac{g^2}{m^2+(\vec{p}-\vec{p}')} = (*)$$

$$(*) = \frac{g^2}{m^2}\left[1 - \frac{(\vec{p}-\vec{p}')^2}{m^2} + \frac{(\vec{p}-\vec{p}')^4}{m^4} + \dots\right]$$

$\rightarrow$ valid for $|\vec{p}-\vec{p}'|^2 << m^2$

At long distances we don't know if two particles exchange a boson



(short) $\to$ (long)

$$V(\vec{q}) = \frac{g^2}{m^2 + |\vec{q}|^2} \longrightarrow C_0 + C_2 \vec{q}^{\,2} + \cdots$$

(potential w/ details) $\to$ (generic potential)

$$\Rightarrow \left[V_c = C_0 + C_2 \vec{q}^{\,2} + C_4 \vec{q}^{\,4} + \ldots \right]$$

$\hookrightarrow$ any potential we can imagine will look like a power series in momenta at low energies / long distances

So for big bound states(*)
we can use the generic potential

$$V_c(\vec{q}) = C_0 + C_2\vec{q}^2 + \dots$$

(*) -> 技术名词: shallow
bound state

And of course we simplify it as:

$$\boxed{V_c^{LO}(\vec{q}) = C_0}$$

Our Leading Order (LO)
approximation to most
two-body problems

But this potential has a problem:

$$V_c^{LO}(\bar{q}) = C_0 \longrightarrow V_c^{LO}(\vec{r}) = C_0\, \delta^{(3)}(\vec{r})$$

$$\left[\int \frac{d^3\bar{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} = \delta^{(3)}(\vec{r}) \right]$$

→ this is a Dirac-delta in 3-dimensions

$$\Downarrow$$

This type of potentials can't be solved

Reminder:

$$\delta(x) = \begin{cases} \infty & x = 0 \\ 0 & x \neq 0 \end{cases} \quad / \int_{-\infty}^{\infty} \delta(x)\, dx = 1$$

$$\text{and } \int_{-\infty}^{\infty} \delta(x) f(x)\, dx = f(0)$$

New concept: $\underbrace{\text{REGULARIZATION}}_{\text{make something finite}}$

$$V_c^{LO}(\vec{q}) = C_0 \xrightarrow{\boxed{\text{REGULARIZE}}}$$

$$V_c^{LO}(\vec{q};\Lambda) = C_0 \, f\left(\frac{|\vec{q}|}{\Lambda}\right)$$

$\Lambda \to$ cutoff

$$f(x \to 0) \to 1$$
$$f(x \to \infty) \to 0$$

Example: $V_c^{LO} = C_0 \, e^{-\left(\frac{q}{\Lambda}\right)^2}$

1) $\Lambda \neq \infty \Rightarrow V_c^{LO}$ is regular

2) $\lim_{\Lambda \to \infty} V_c^{LO} = C_0$

So we can use the generic potential

$$V_c^{LO}(\vec{q}) = C_0 \, e^{-\left(\frac{q^2}{\Lambda^2}\right)}$$

$$\rightarrow V_c^{LO}(\vec{r}) = C_0 \, \frac{\Lambda^3}{8\pi^{3/2}} \, e^{-\frac{1}{4}\Lambda^2 r^2}$$

If $\Lambda \gg \gamma$, the bound state should not notice the difference

$\longrightarrow$ $\boxed{\text{BUT WAIT !?}}$

$\Downarrow$

What happens w/ the cutoff ?

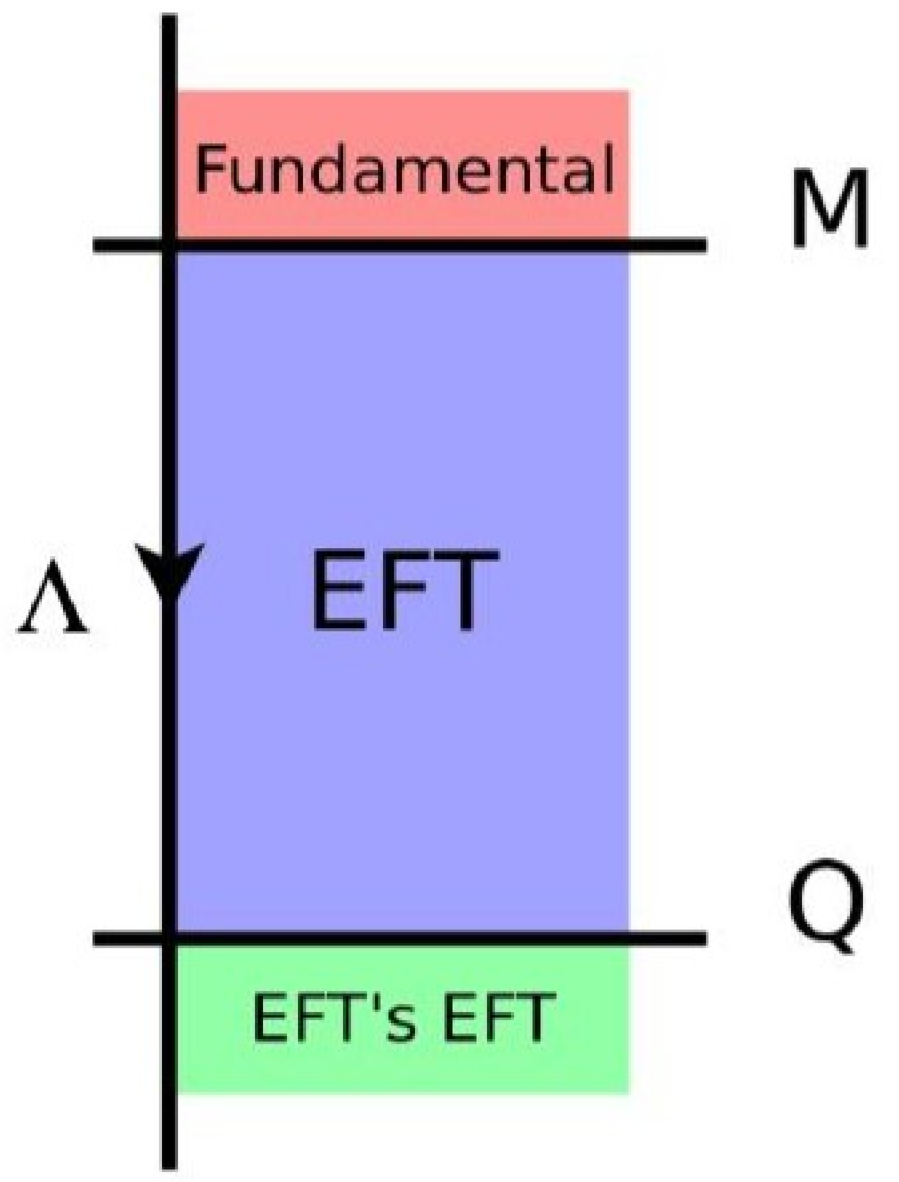
REGULARIZATION :

→ we included Λ to make calculations finite

But what we really want is...

$\boxed{\text{RENORMALIZATION}}$

→ Cutoff independence



Physics is unique, but choice of theory depends on resolution Λ:

- $\Lambda \geq M$: Fundamental
- $M \geq \Lambda \geq Q$: EFT

For equivalent descriptions:

$$\boxed{\frac{d}{d\Lambda}\langle\Psi|\mathcal{O}|\Psi\rangle = 0}$$

Renormalization group invariance

↳ Remember this?

How do we do it?

1) $V_c^{LO} = \underbrace{C_0(\Lambda)}_{\displaystyle C_0 = C_0(\Lambda)} \frac{\Lambda^3}{8\pi^{3/2}} e^{-\frac{1}{4}\Lambda^2 r^2}$

$\Rightarrow$ [running coupling]

2) $V_c^{LO} \longrightarrow \gamma = \gamma(C_0)$

$= \gamma(C_0(\Lambda))$

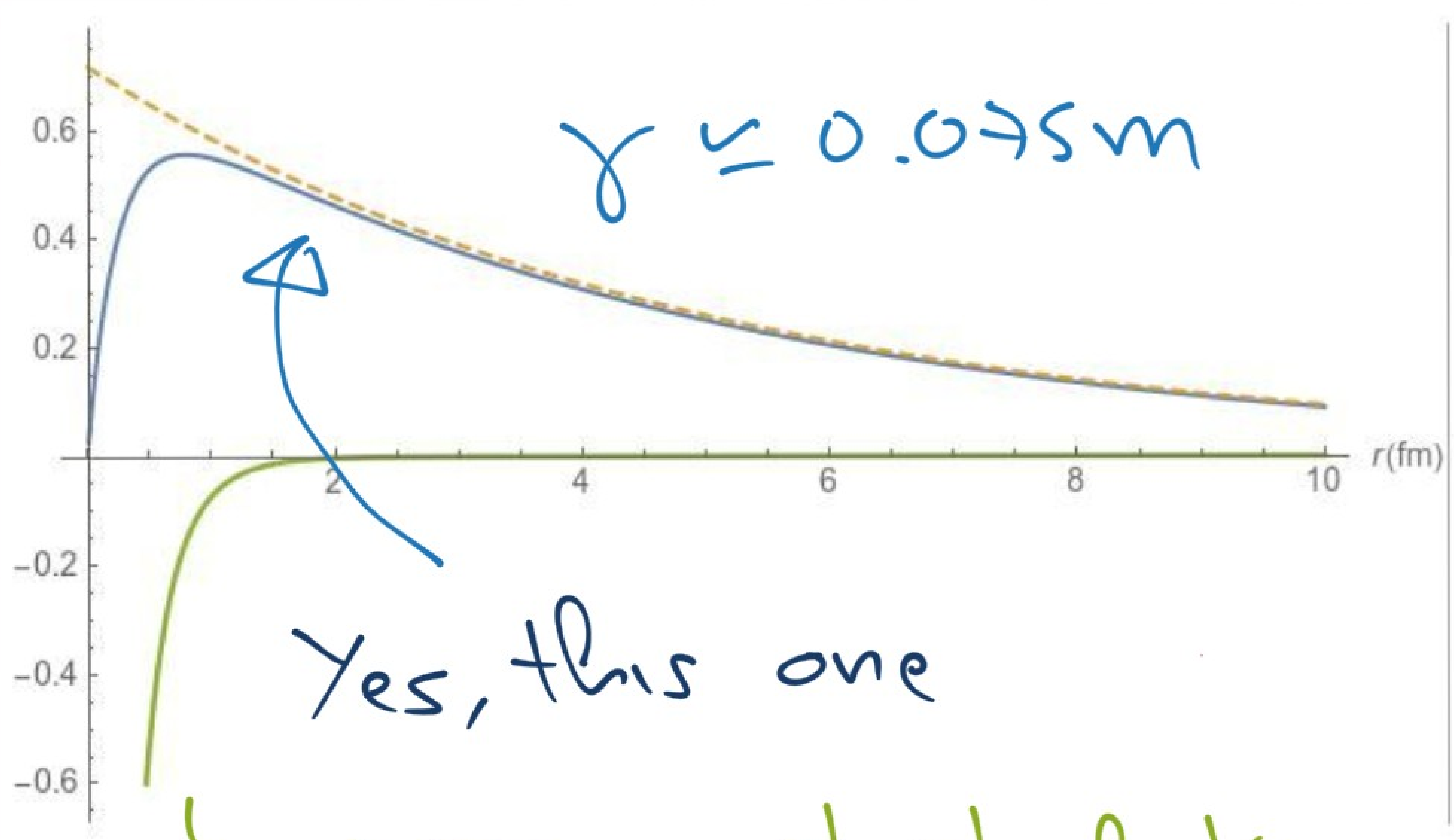
$$\Rightarrow \boxed{\frac{d}{d\Lambda}\gamma(\Lambda) = 0}$$

RG Equation

Equivalent to $\boxed{\frac{d}{d\Lambda}\langle \Psi | H | \Psi \rangle = 0}$

Let's see how it does look like for the solution of

$$V_Y(r) = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}$$



$\gamma \simeq 0.075\, m$

Yes, this one

$\rightarrow$ True potential V_Y

$\Rightarrow$ Now let's reproduce this bound state w/ V_c^{LO}

$$V_c^{LO}(r) = C_0(\Lambda)\frac{\Lambda^3}{8\pi^{3/2}} e^{-\frac{\Lambda^2 r^2}{4}}$$

Let's do it: 1) Choose Λ

2) Fit r_0 to γ

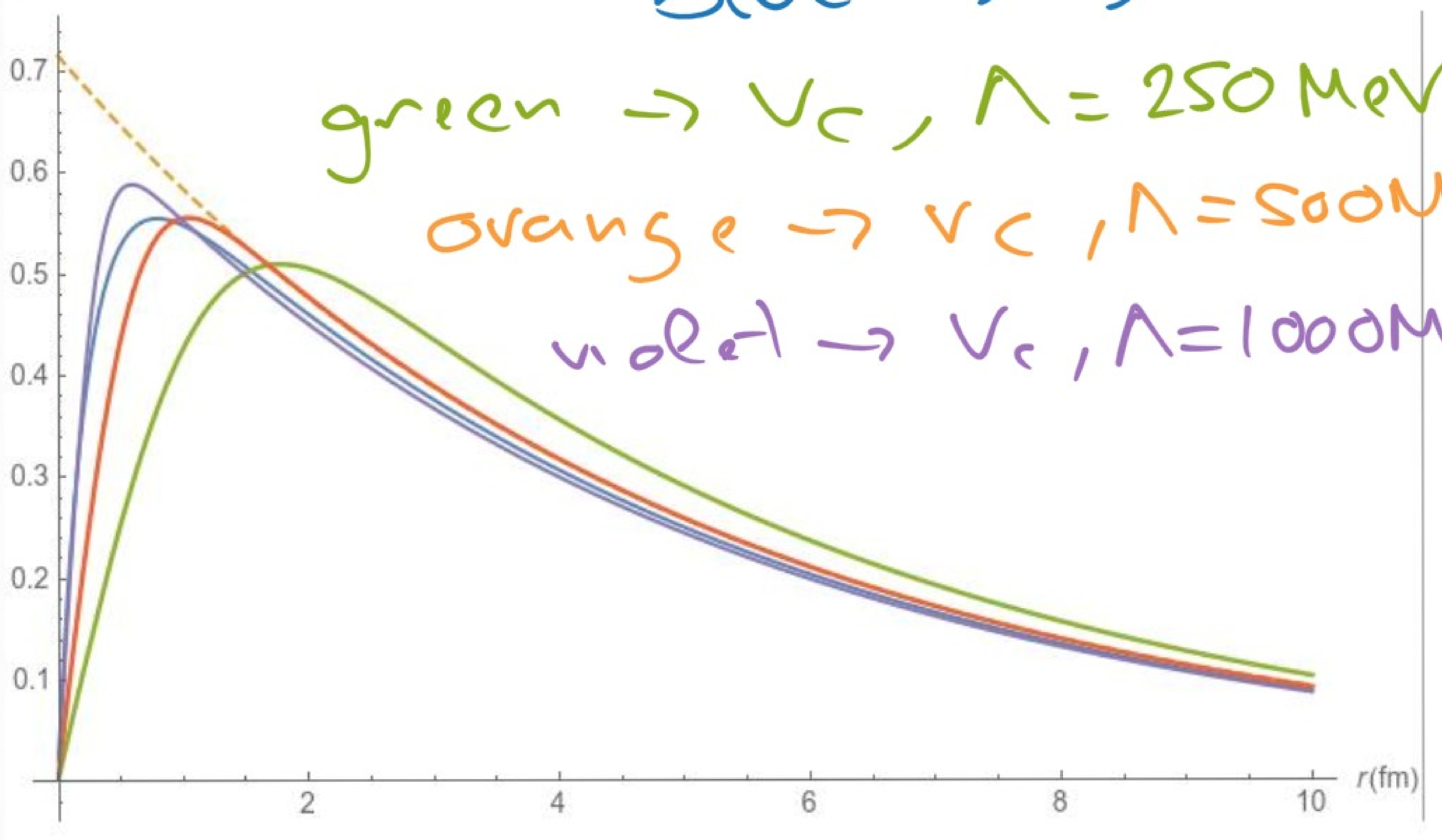
Result:

blue $\to V_\gamma$

green $\to V_c$, $\Lambda = 250$ MeV

orange $\to V_c$, $\Lambda = 500$ MeV

violet $\to V_c$, $\Lambda = 1000$ MeV



→ They are very similar to the original potential

→ $\Lambda \gtrsim m$ for best results

($m = 500$ MeV)

→ for $\Lambda \to \infty$ we recover

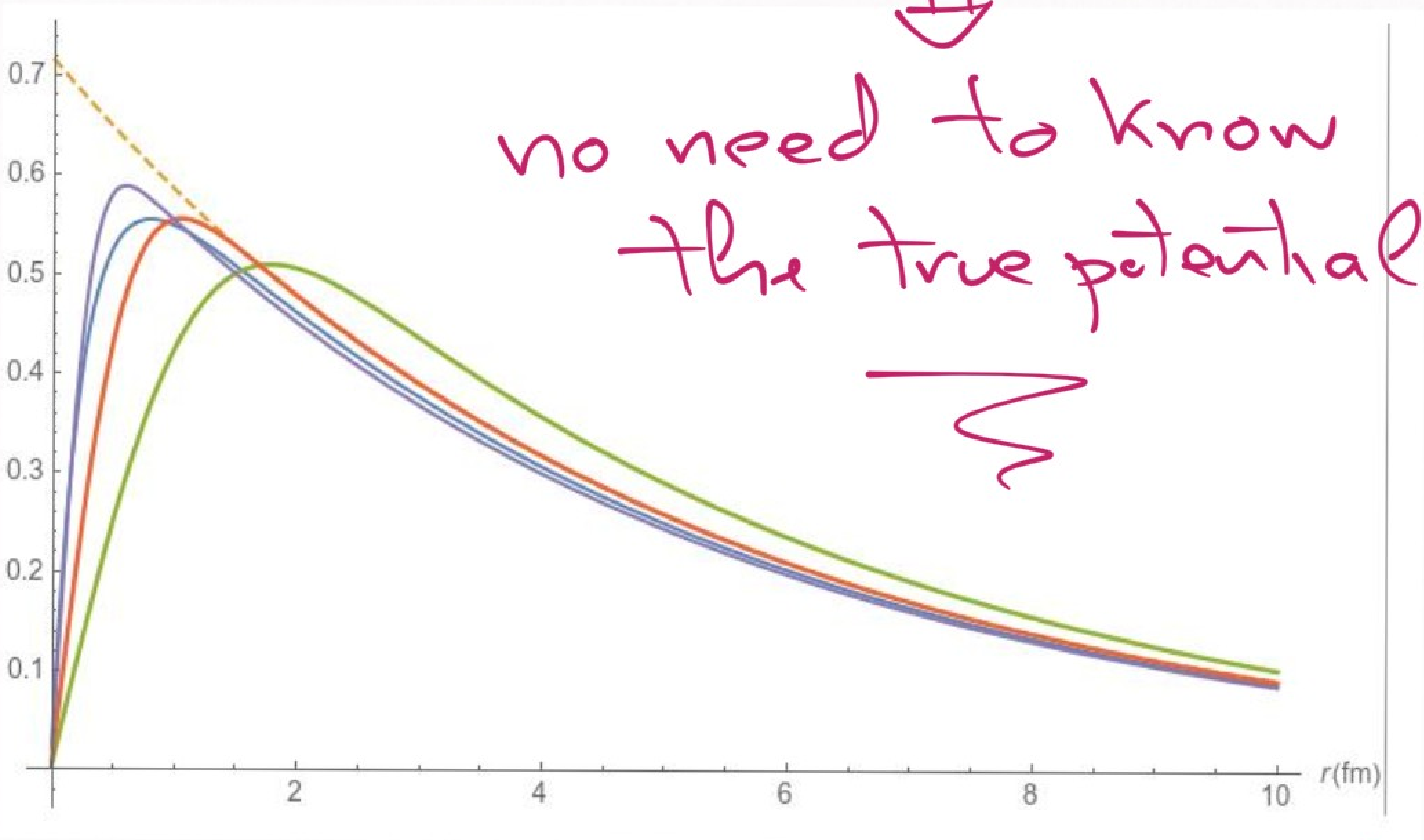
$u(r) = A_s e^{-\gamma r}$

All the wave functions similar

$\Downarrow$

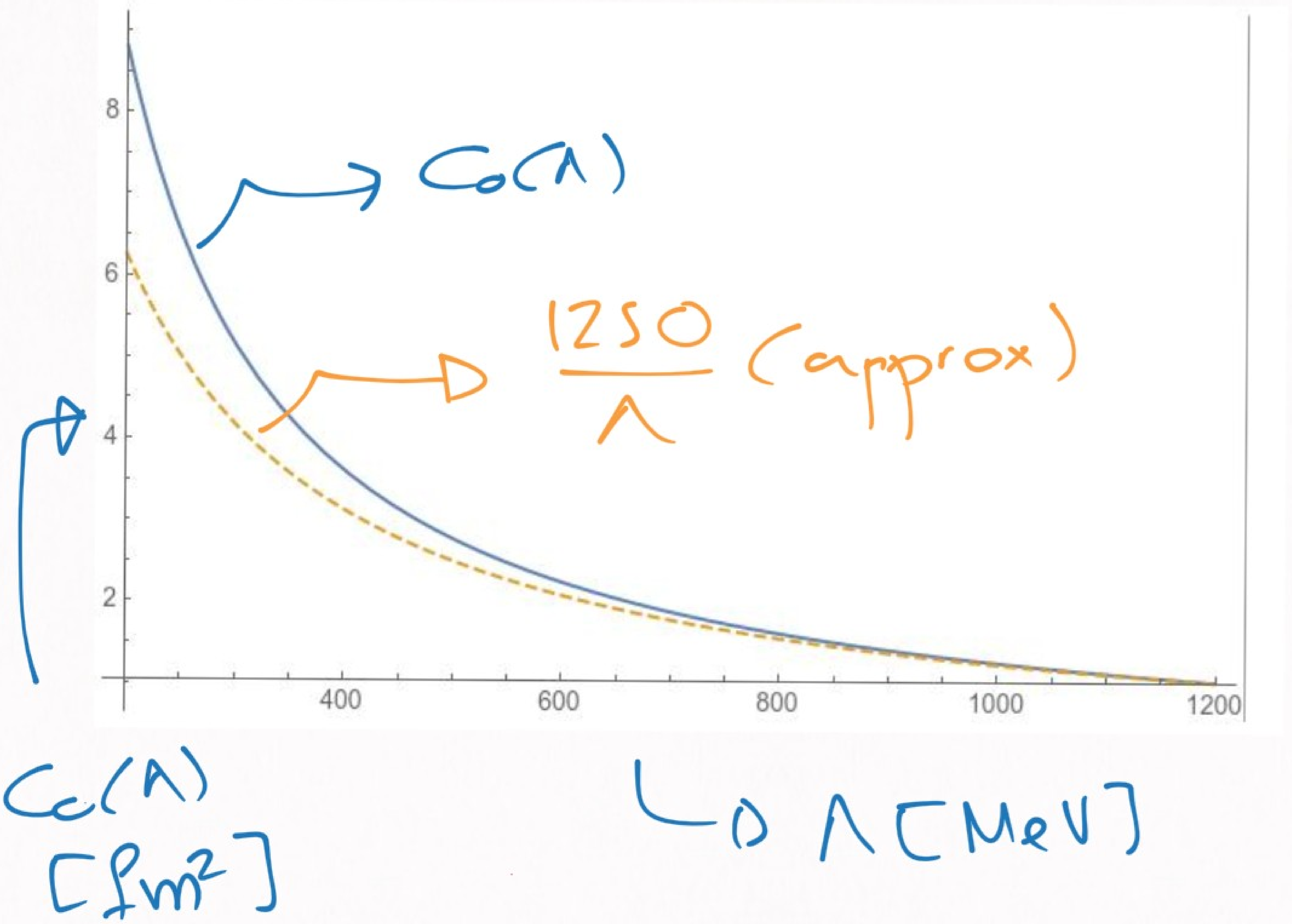
no need to know the true potential

$\Rightarrow V_c^{LO}$ is an EFT for $|\cdots)$



$\rightarrow$ This is how EFT works

Remember $\Rightarrow$ $C_0 = C_0(\Lambda)$



$$\Rightarrow C_0(\Lambda) \sim \frac{1}{\Lambda}$$

$$\Rightarrow \underbrace{\frac{d}{d\Lambda}\left[\Lambda C_0(\Lambda)\right] \sim 0}_{\text{RGE for } C_0 \text{ in this case}}$$

REVIEW :

1) No need to know "V_{TRUE}"

→ only need effective V

e.g. $V^{LO}_{EFT} = C_0$

2) Regularize effective V

$$V^{LO}_{EFT} = C_0 e^{-(q^2/\Lambda^2)}$$

3) Renormalize effective V

$$C_0 \to C_0(\Lambda)$$

$$\frac{d}{d\Lambda} \langle \Psi | \hat{O} | \Psi \rangle = 0$$

4) Do calculations, describe the system, publish paper

ADDITIONAL NOTES :

→ there are countless ways to regularize

$(*)\ V(\bar{q}) = C_0(\Lambda)\, e^{-q^2/\Lambda^2}$

$V(\bar{q}) = C_0(\Lambda) \dfrac{\Lambda^2}{\Lambda^2 + \bar{q}^2}$

p-space

$V(r) = C_0(R_c) \dfrac{\delta(r - R_c)}{4\pi R_c^2}$

$(*)\ V(r) = C_0(R_c) \dfrac{e^{-(r/R_c)^2}}{R_c^3 \pi^{3/2}}$

r-space

→ You can try & experiment yourself

(*) → Equivalent for $\Lambda R_c = 2$

RECOMMENDATIONS:

1)

3. **Effective field theory of short range forces**
U. van Kolck (Caltech, Kellogg Lab & Washington U., Seattle). Aug 1998. 38 pp.
Published in **Nucl.Phys. A645 (1999) 273-302**
KRL-MAP-230, NT-UW-98-01
DOI: 10.1016/S0375-9474(98)00612-5
e-Print: **nucl-th/9808007** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 331 records 250+

about $V_c = C_0 + C_2 q^2 + \cdots$ potentials in EFT, from the EFT 泰斗

2)

1. **How to renormalize the Schrodinger equation**
G.P. Lepage (Cornell U., LNS). Feb 1997. 46 pp.
Lectures given at Conference: C97-01-26.2, p.135-180 Proceedings
e-Print: **nucl-th/9706029** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
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a classic, but... Lepage is a practical guy who is not interested in $\Lambda \to \infty$, and argues against it

Just try to understand why he is wrong (or right) while you read it (critical thinking)