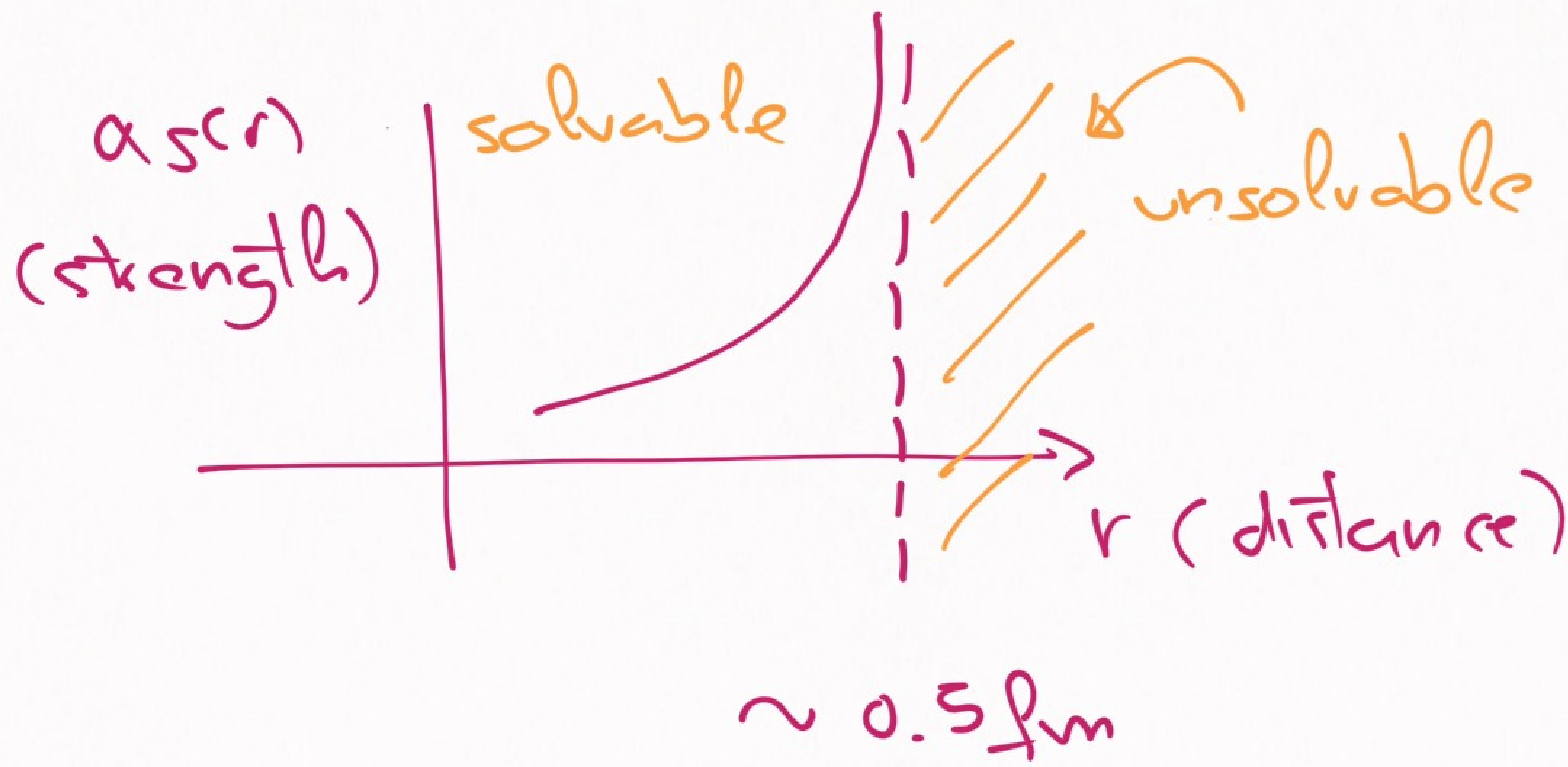
# Nuclear Physics ⑦

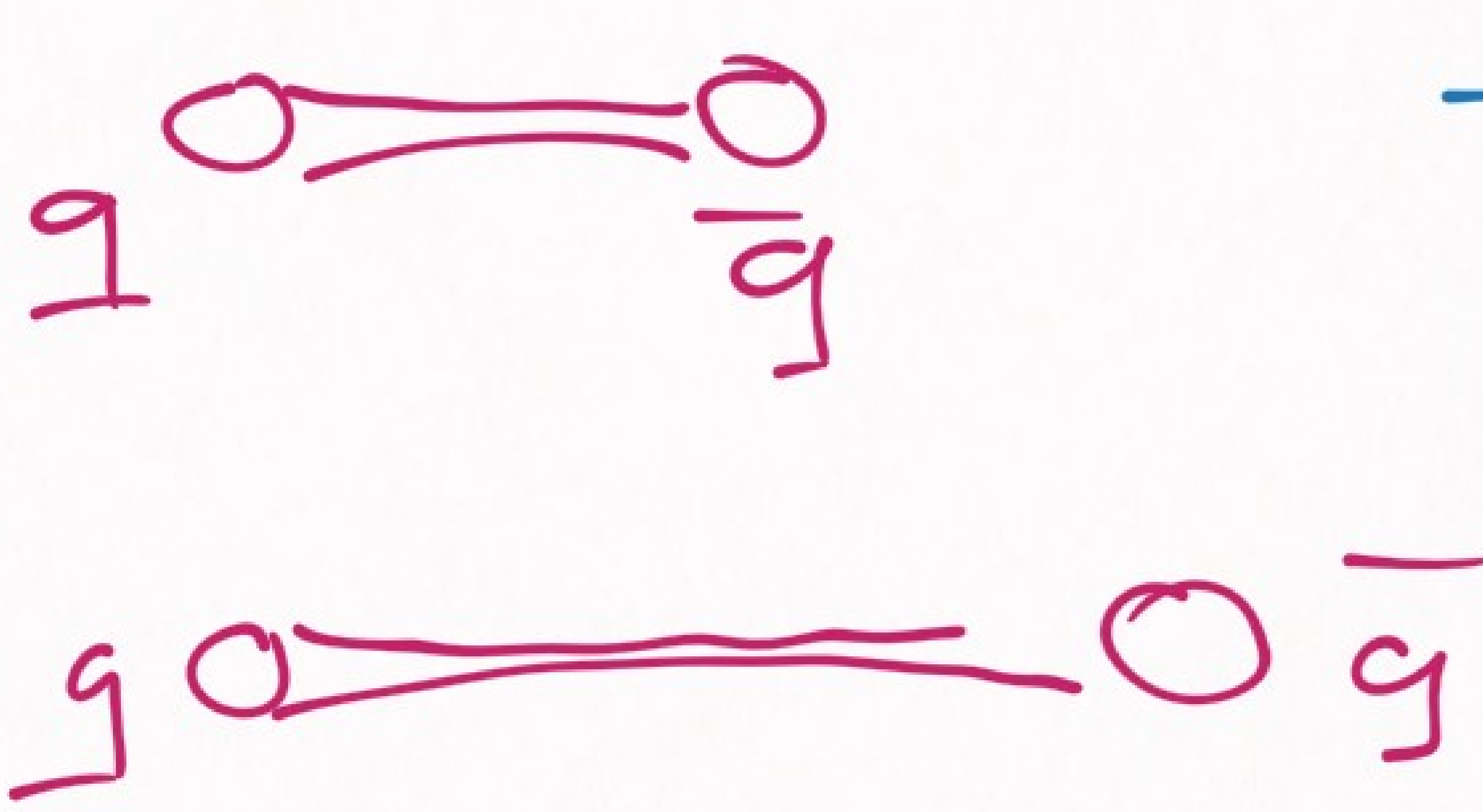
QCD can't be solved analytically

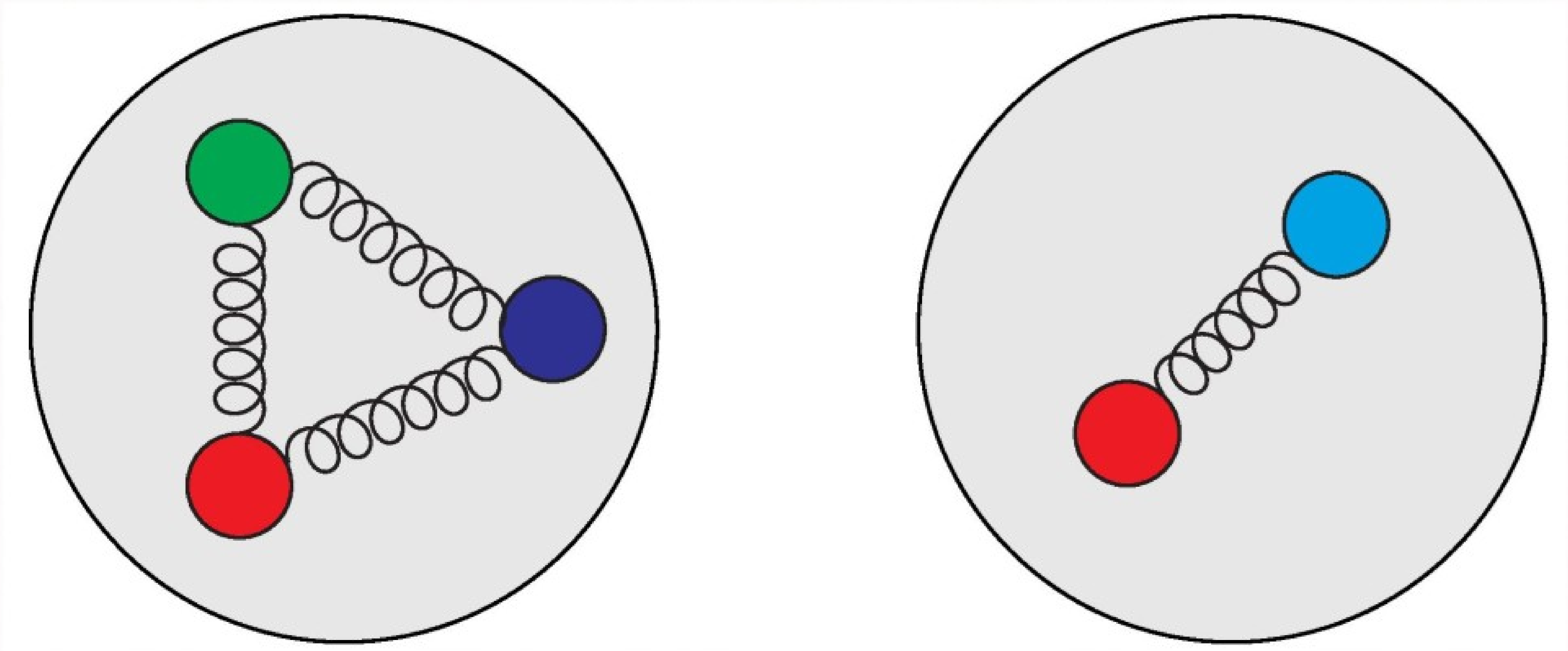
What are the alternatives?

RECAP: QCD has asymptotic freedom & confinement




confinement → no free quarks
↓
this happens when you try to separate quarks



Baryon → Proton
$r_p \sim 0.85\,fm$

Meson → Pion
$r_\pi \sim 0.65\,fm$

↓

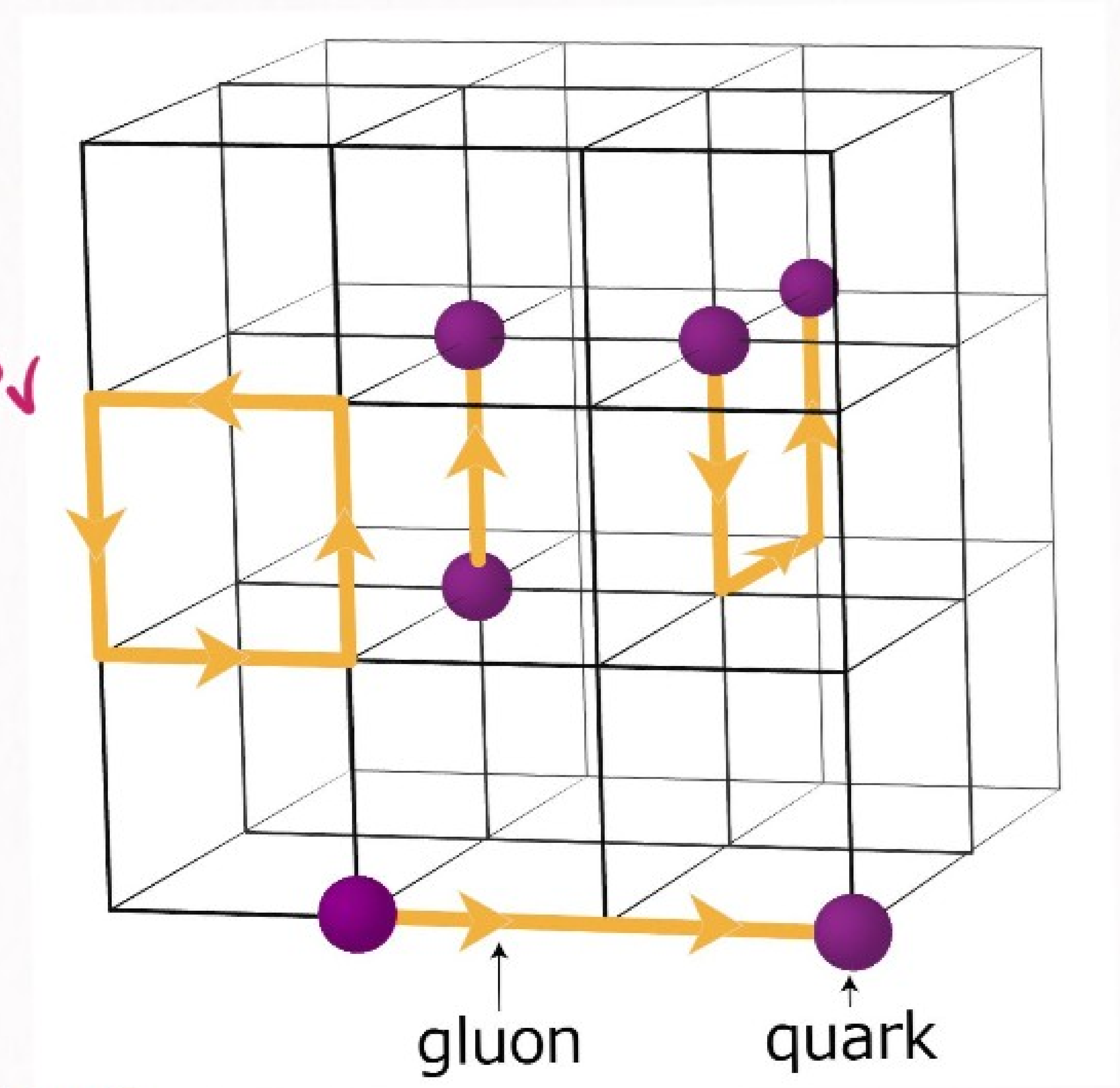
Too large for QCD

↓

Alternatives ?

Two possibilities are :

1) Lattice QCD



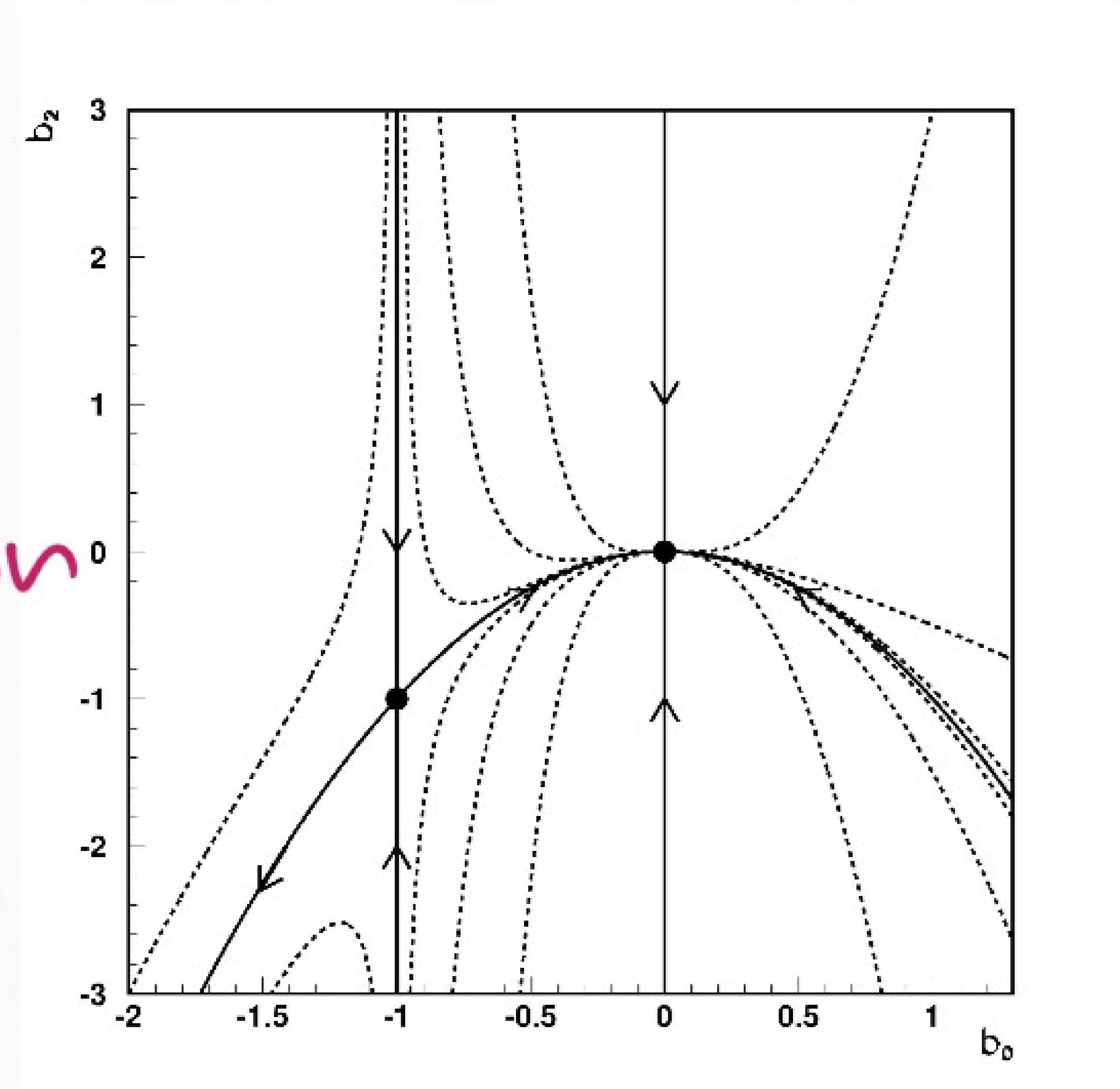

Use supercomputer to solve QCD

↓

I don't have a supercomputer

↓

2) Effective Field Theory




Use renormalization group analysis to indirectly solve QCD

↳ My choice: I only have pen & paper & a tablet

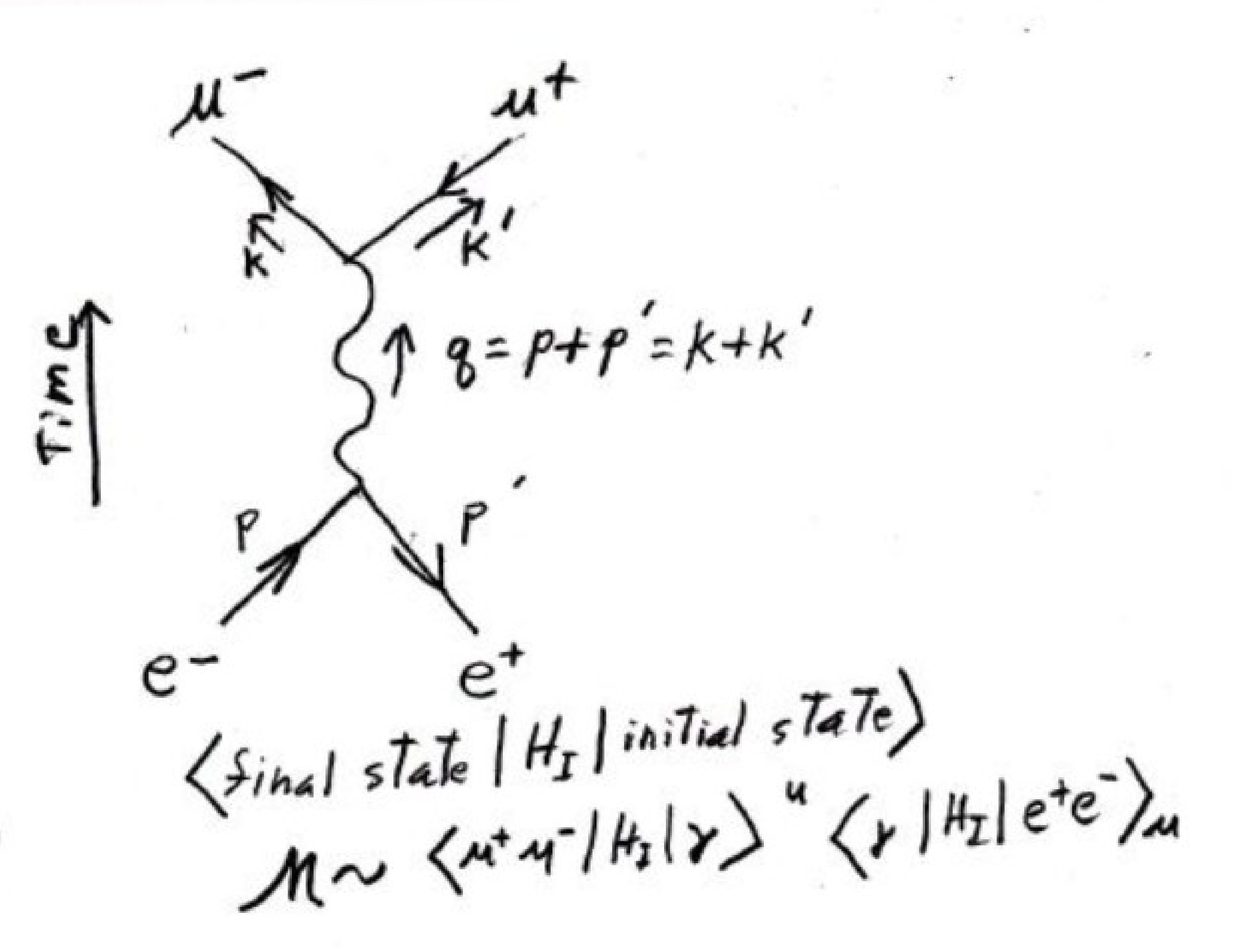
# EFFECTIVE FIELD THEORIES (EFTs)

What are they?

→ Let's begin w/ RENORMALIZATION

1) ONCE UPON A TIME...

→ Actually, in Pocono, 1948




Feynman & Schwinger present really strange methods to solve the infinities in QED

# RENORMALIZATION

→ a set of arcane methods to remove infinities

Example : harmonic series

$$H(n) = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$$

↳ $\lim_{n \to \infty} H(n) \to \infty \quad (\mathcal{O}(\log n))$

↳ But if we use $\zeta$-regularization

$$\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \dots$$

↳ Riemann zeta function

$$\lim_{n \to \infty} H(n) = \zeta(1) \,, \quad \zeta(1) = -\frac{1}{12}$$

$$\Rightarrow \left[ 1 + \frac{1}{2} + \frac{1}{3} + \dots = -\frac{1}{12} \right]$$

Feynman described it as:

我们为求出n和j所玩的壳层游戏，在专业上叫做“重正化”（renormalization）。但是，不管这个词听来多聪明，我却说这个过程是蠢笨的！求助于这类戏法妨碍了我们去证明量子电动力学在数学上的自洽性（self-consistent）。令人不解的是，尽管人们用了各种办法，这个理论至今仍未被证实是自洽的；我猜想，重正化在数学上是不合法的。我们还没有一种好的数学方法描述量子电动力学，这是肯定的——像这样描述n、j同m、e之间关系的语言不是好的数学。[23]

(from QED: the strange theory of light & matter / QED：光和物质的奇妙理论)

- So it appears that the only things that depend on the small distances between coupling points are the values for n and j-theoretical numbers that are not directly obseroable any- way; everything else, which can be observed, seems not to be affected. The shell game that we play to find n and j is technically called "renormalization." But no matter how clever the word, it is what I would call a dippy process! Having to resort to such hocus-pocus has prevented us from proving that the theory of quantum electrodynamics is mathematically self-consistent. It's surprising that the theory still hasn't been proved self-consistent one way or the other by now; I suspect that renormalization is not mathematically legitimate. What is certain is that we do not have a good mathematical way to describe the theory of quantum electrodynamics: such a bunch of words to describe the connection between n and j and m and e is not good mathematics.
    - Richard Feynman, *QED: The Strange Theory of Light and Matter* (1985), Chap. 4. Loose Ends

↑ ORIGINAL FROM WIKIQUOTES

But after 70 years our understanding of renormalization has vastly improved

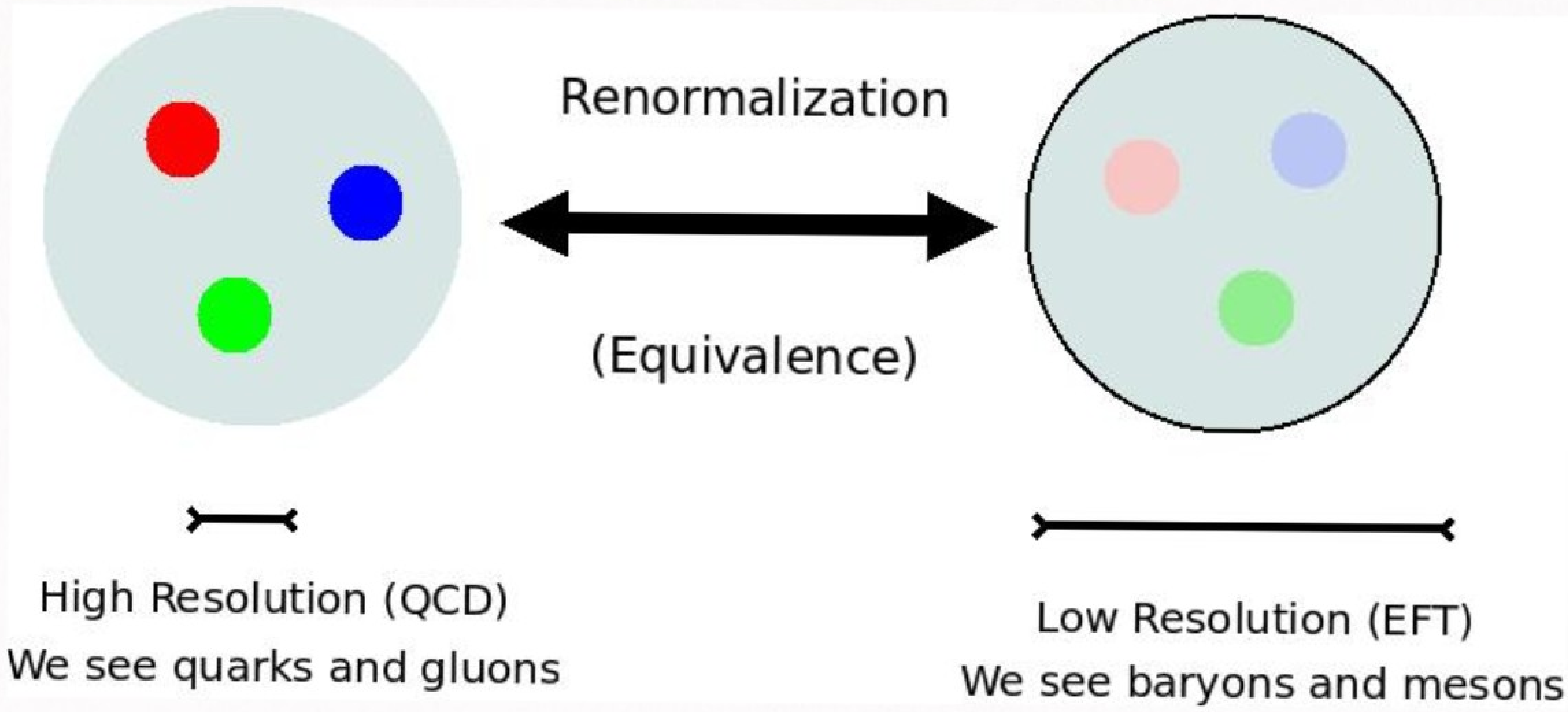
— ⊗ —

BASIC IDEA:

| |
|---|
| Physics at long-distances does not depend on short-distance details |

⇓

Renormalization is just the mathematical implementation of this idea




↳ This is how it looks for hadron & nuclear physics

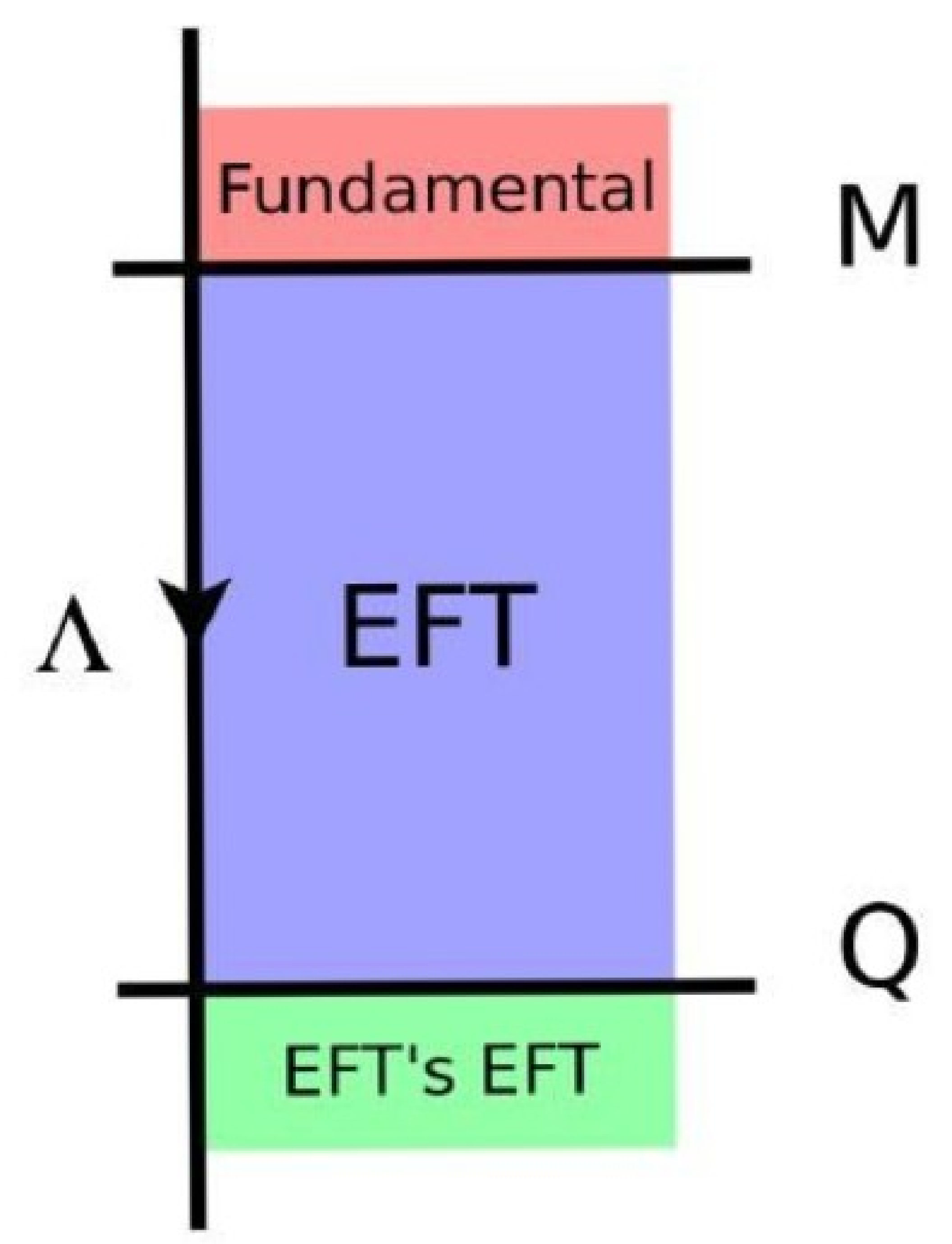
↳ RENORMALIZATION:

connects in a rigorous way the different views of a problem

Quarks & gluons ⟺ Baryons & mesons

RELATED CONCEPT :

# Effective Field Theories (EFTs)




Physics is unique, but choice of theory depends on resolution Λ:

- $\Lambda \geq M$: Fundamental
- $M \geq \Lambda \geq Q$: EFT

For equivalent descriptions:

$$\frac{d}{d\Lambda}\langle \Psi | \mathcal{O} | \Psi \rangle = 0$$

Renormalization group invariance

[ Λ → energy scale we are looking at

M → scale of fundamental theory

Q → scale of low energy theory

→ **THE CUTOFF**

FOR QCD:

→ $M \sim 1\,GeV$ ($1/M \sim 0.2\,fm$)

mass of most hadrons

→ $Q \sim m_\pi \sim 140\,MeV$

($1/Q \sim 1.4\,fm$)

pion mass, average separation of nucleons in nuclei

---

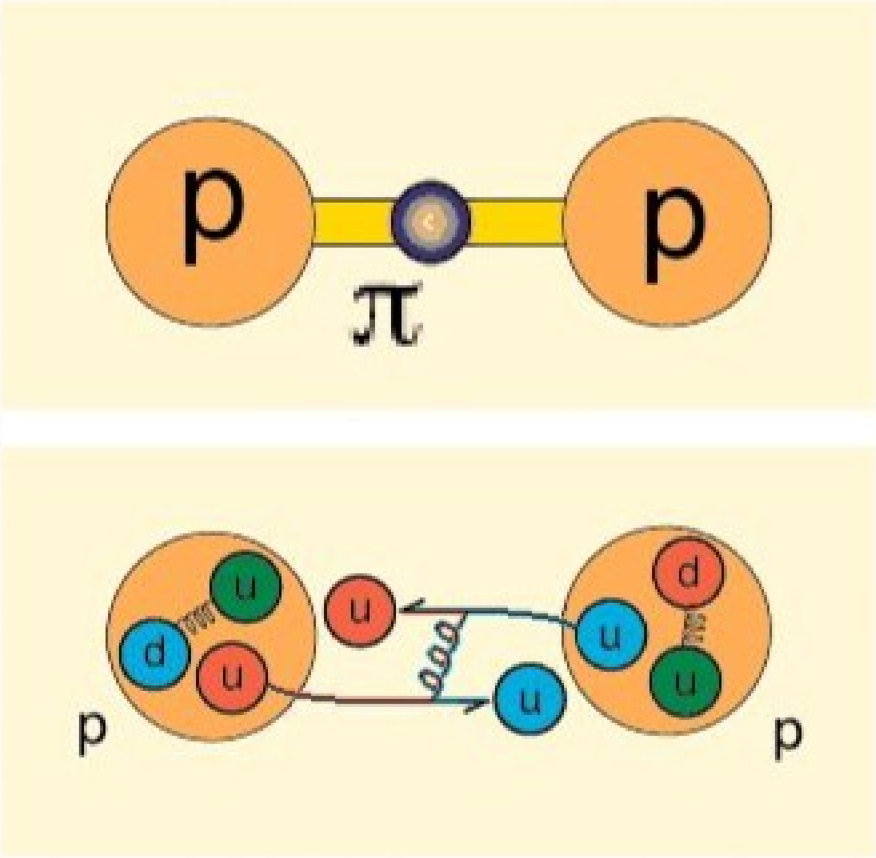
1) $\Lambda > M$ → quark & gluons

2) $\Lambda < M$ → baryons & meson

3) $\Lambda \sim M$ → both descriptions possible

This tells us how to build an EFT:

1) $\Lambda \sim M$ two equivalent descriptions:




baryons & mesons $\longleftrightarrow$ quarks & gluons

2) If $\Lambda \to \infty$ both descriptions will be equivalent if and only if observables $\Lambda$-independent

$$\frac{d}{d\Lambda}\langle\Psi|\hat{O}|\Psi\rangle = 0$$

$\Psi$ → wave function

$\hat{O}$ → Observable

But there is a problem

→ Predictive power

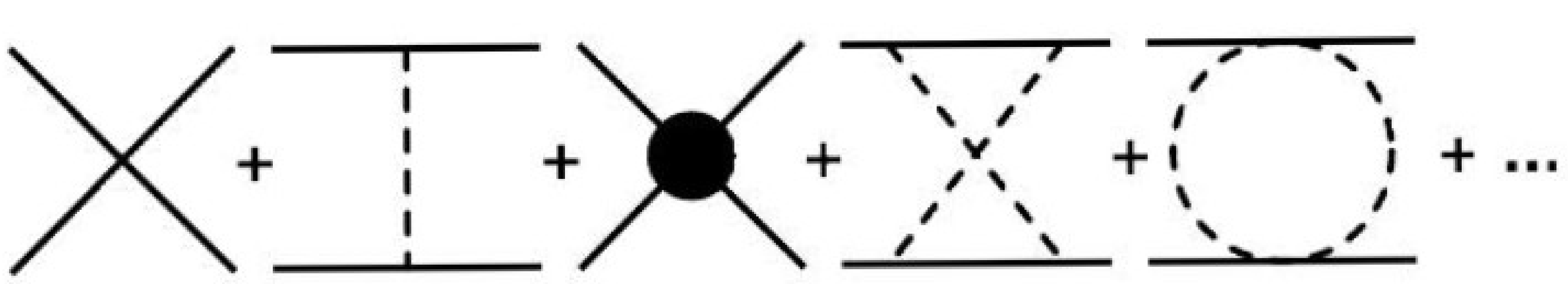
Begin at $\Lambda = M$, two equivalent descriptions

$$\underbrace{\text{quarks \& gluons}}_{\text{high energy}} \iff \underbrace{\text{hadrons } (N, \Delta, \pi)}_{\text{low energy}}$$

The hadron description equivalent if and only if

(1) Include low energy symmetries (particularly **chiral symmetry**)

(2) Consider infinite set of Feynman diagrams consistent with (1)



Problem: **infinite diagrams imply no predictive power**

1) FUNDAMENTAL DESCRIPTION

→ Quark & Gluons → QCD

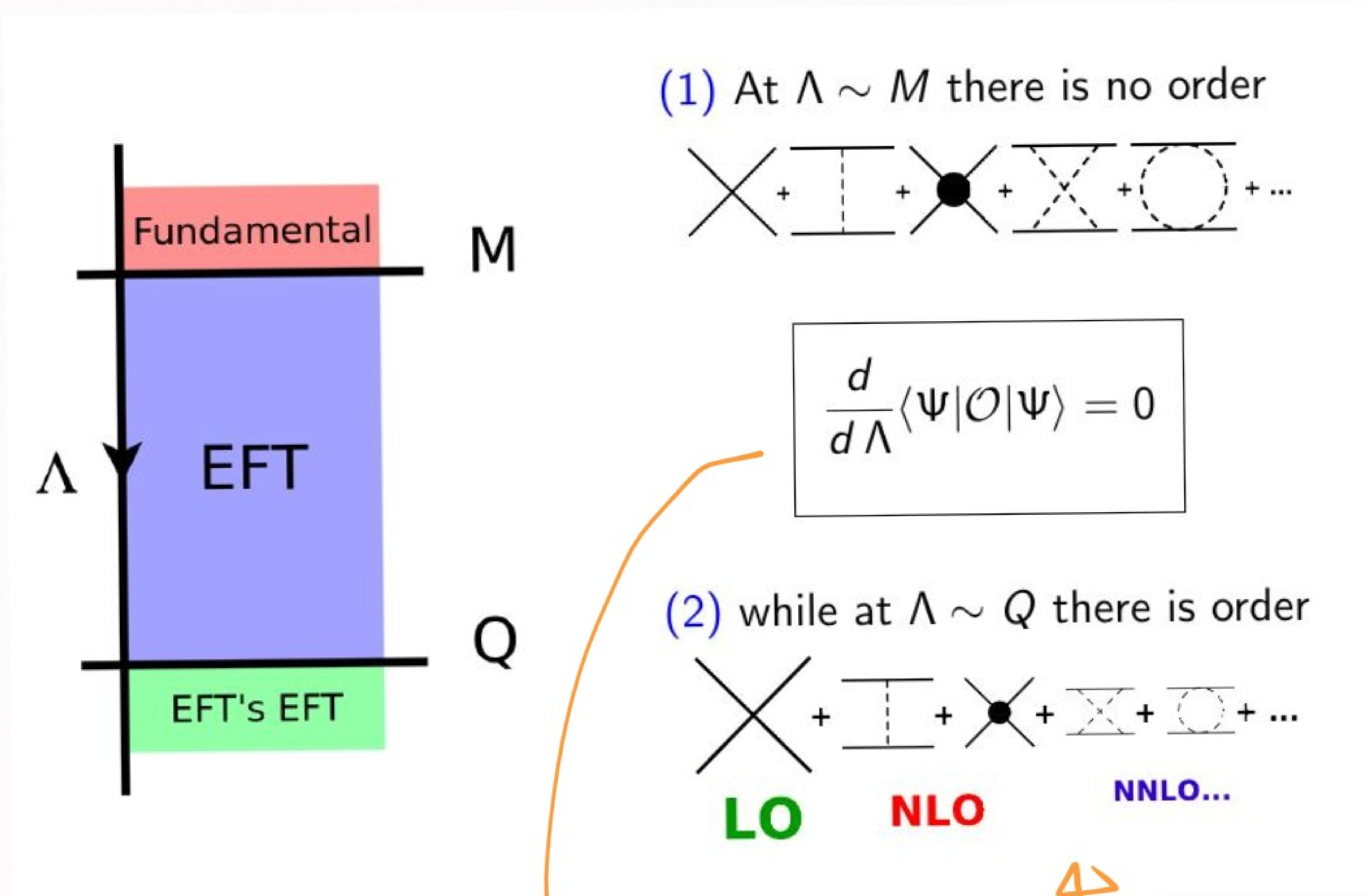
2) EFFECTIVE DESCRIPTION

→ Nucleons & pions

→ Only equivalent if we include all possible interactions (diagrams)

# RECOVERING PREDICTIVE POWER

## → POWER COUNTING




RGA implies that there's order at low energies / large distances

Power counting → some factors are more important than others

→ a bit too abstract,
so let's use an example

## THE WONDERFUL THEORY OF TEACUPS & TEAPOTS

The good life (TM)

& a really good example
of an effective theory

A very innocent question:

Where does the tea cools faster?


(1) The Teapot?

(2) The Teacup?

If your answer was (2):

Congratulations! You were right

↳ BUT WHY?


(1) TEAPOT:

a) Ceramic surface large

b) Exposed surface small

(2) TEACUP: a) small

b) large

HEAT TRANSFER :

[intituively convection faster than conduction]

⇓

[TEACUP COOLS FASTER THAN TEAPOT]

⇓

But we didn't solve any equation describing HEAT TRASFER

↓

Fundamental theory

Effective theory

# FUNDAMENTAL THEORY:

1) Fourier's law of heat conduction

$$\underline{q} = -k \nabla T$$

2) Convection-diffusion eq:

$$\frac{\partial c}{\partial t} = \nabla \cdot (D \nabla c) - \nabla \cdot (\underline{v} c) + R$$

(taken from Wikipedia)

3) Use your compiler and solve it for teapots & teacups

STRAIGHTFORWARD

BUT...

[I'm a really lazy guy]

EFFECTIVE THEORY → The smart way

1) Newton's law of cooling

$$T = T_0 e^{-\lambda(t - t_0)}$$

$T_0, t_0$ → initial temperature & time

$T, t$ → final temperature & time

→ so I want to compute $\lambda$

2) Find the relevant degrees of freedom

→ Exposed surface $S$

Ceramic surface $\Sigma$

## 3) Propose a power counting

S more important than Σ

⇒ expand in powers of

$$x = \left(\frac{S}{f\Sigma}\right)$$

f: correction factor b/c ceramic cools slower

## 4) Write down the theory

$$\lambda = S\left(c_0 + c_1 x + c_2 x^2 + \dots\right)$$

Low Energy Constants (LECs)

it's a power series!

(that is why we call it power counting)

5) Choose the accuracy we want & cut the series

$\mathcal{O}(x^0)$ : $\lambda = c_0 S$

↳ also called "Leading Order" (LO)

$\mathcal{O}(x^1)$ : $\lambda = c_0 S + c_1 S x$

↳ also called "Next to Leading Order" (NLO)

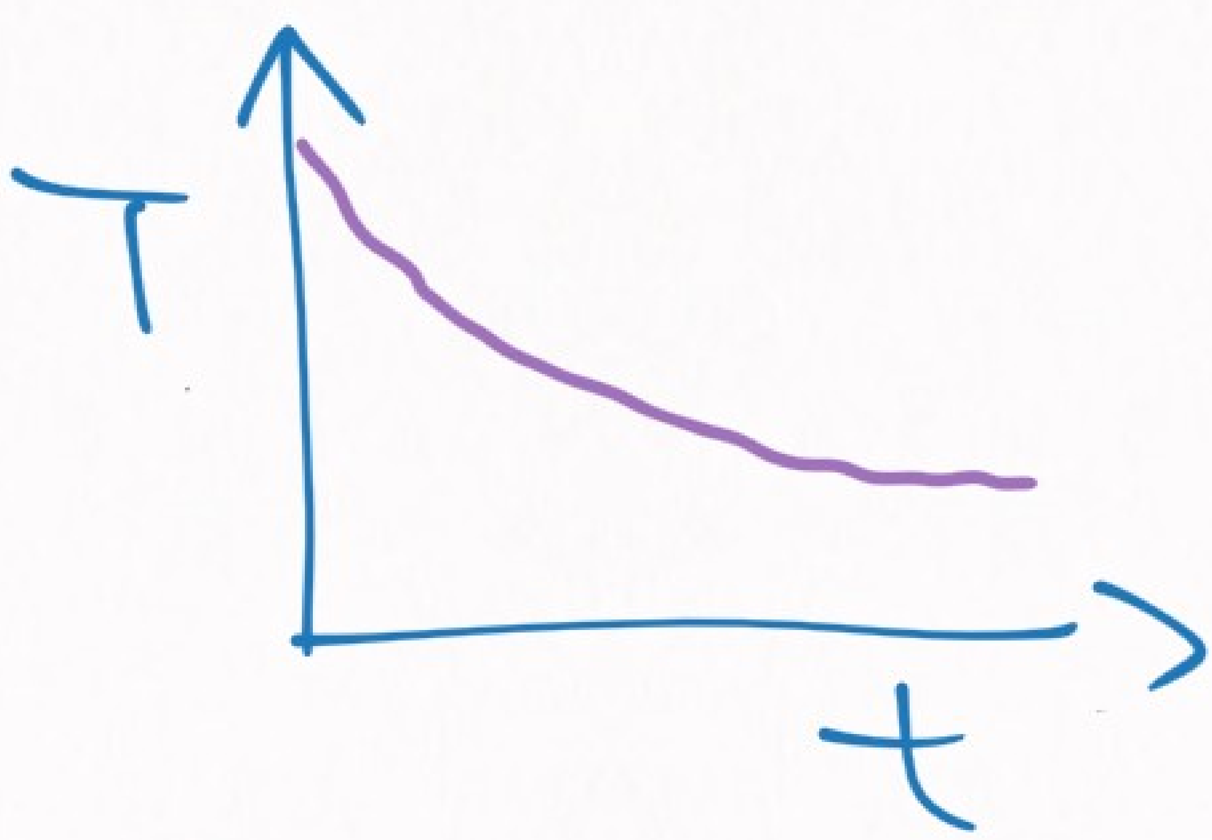
$\mathcal{O}(x^2)$ : $\lambda = S(c_0 + c_1 x + c_2 x^2)$

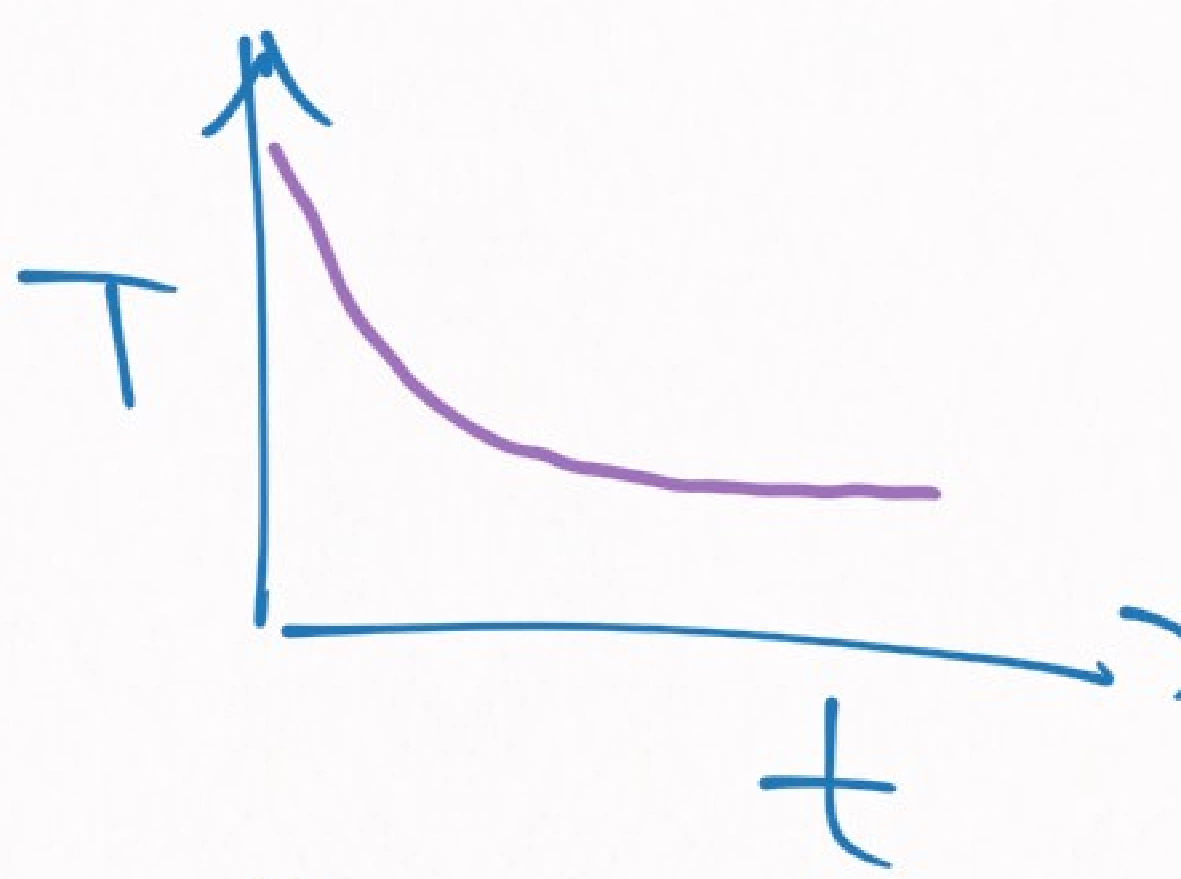
↳ NNLO or $N^2LO$

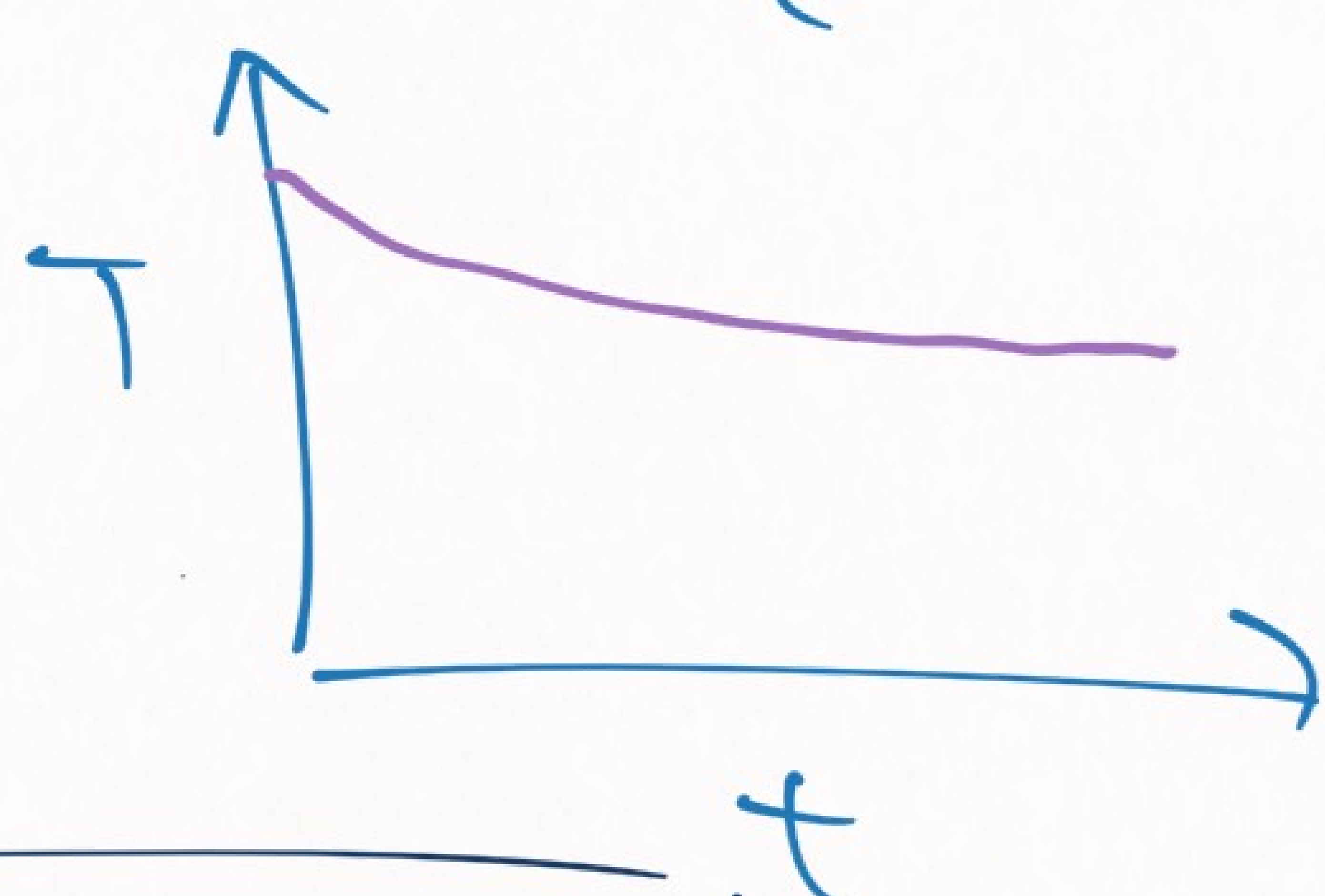
More terms ⇒ more accuracy (but also more difficult)

## 6) Fit the LECs ($c_0, c_1, c_2, \dots$) to experimental data

→ my instruments




⇒ **Fit the LECs**

RECAP:

1) Some dynamics (cooling's law / QM / QFT)

2) Degrees of freedom (surfaces' types / particles ...)

3) Power counting

4) Write down the theory

↳ LECs

5) Choose accuracy

↳ LO, NLO, $N^2LO$, ...

6) Fit to experimental data

— ⊗ —

RELATIVELY SIMPLE

BUT THERE IS MORE

→ ∃ many power countings

Example: the teapot theory


$$\lambda_{Pot} = \Sigma\left(p_0 + \frac{p_1}{x} + \frac{p_2}{x^2} + \dots\right)$$

$$= \Sigma\left(p_0 + p_1 y + p_2 y^2 + \dots\right)$$

$$y = \frac{\ell\Sigma}{S}\ , \quad y < 1$$

Power counting not unique:

"Cup" power counting

$$\lambda_c = \Sigma\left(c_0 + c_1 x + c_2 x^2 + \dots\right)$$

(valid for $x < 1$)

"Pot" power counting

$$\lambda_p = \Sigma\left(p_0 + \frac{p_1}{x} + \frac{p_2}{x^2} + \dots\right)$$

(valid for $x > 1$)

In RG language we call these:

UNIVERSALITY CLASSES

OR

INFRARED FIXED POINTS

→ they are the infrared ($\Lambda \to 0$) solutions of the RG (Renormalization Group)

More about RG language:

$\Lambda \to 0$ INFRARED $(r \to \infty)$

$\Lambda \to \infty$ ULTRAVIOLET $(r \to 0)$

Congratulations!

Now you know how to renormalize your tea set!

$\Rightarrow$ RENORMALIZED!

NEXT → We can try to renormalize something related w/ nuclear physics

— ⊗ —

A few recommendations:

1) The paper that gave us the RG

2. **The Renormalization group and the epsilon expansion**
(2700) K.G. Wilson (Princeton, Inst. Advanced Study & Cornell U., LNS), John B. Kogut
Published in **Phys.Rept. 12 (1974) 75-199**
DOI: 10.1016/0370-1573(74)90023-4
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 2700 records 1000+

↳ requires solid state physics background

2) A wonderful exposition:

4. **Renormalization and Effective Lagrangians**
Joseph Polchinski (Harvard U.). Apr 1983. 27 pp.
Published in **Nucl.Phys. B231 (1984) 269-295**
HUTP-83-A018
DOI: 10.1016/0550-3213(84)90287-6
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
KEK scanned document
Detailed record - Cited by 1120 records 1000+

↳ requires QFT & $\phi^4$ theory

3) This one is awesome & easy

1. **Building light nuclei from neutrons, protons, and pions**
(45) Daniel R. Phillips (Ohio U.). Mar 2002. 54 pp.
Published in **Czech.J.Phys. 52 (2002) B49**
DOI: 10.1007/s10582-002-0079-z
To appear in the proceedings of Conference: C01-07-09.13 Proceedings
e-Print: **nucl-th/0203040** | **PDF**

References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
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4) A bit more difficult, but worth a try

3. **A Renormalization group treatment of two-body scattering**
(196) Michael C. Birse, Judith A. McGovern, Keith G. Richardson (Manchester U.). Jul 1998. 4 pp.
Published in **Phys.Lett. B464 (1999) 169-176**
MC-TH-98-11
DOI: 10.1016/S0370-2693(99)00991-0
e-Print: **hep-ph/9807302** | **PDF**

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5) A classic of nuclear EFT

1. **Effective field theory for few nucleon systems**
(618) Paulo F. Bedaque (LBL, Berkeley), Ubirajara van Kolck (Arizona U. & RIKEN BNL). Mar 2002. 55 pp.
Published in **Ann.Rev.Nucl.Part.Sci. 52 (2002) 339-396**
DOI: 10.1146/annurev.nucl.52.050102.090637
e-Print: **nucl-th/0203055** | PDF

References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service

Detailed record - Cited by 618 records 500+

→ [the big boss]

6) Even I have a short review

23. **Power Counting and Wilsonian Renormalization in Nuclear Effective Field Theory**
(28) Manuel Pavon Valderrama (Beihang U. & Orsay, IPN). Apr 5, 2016. 40 pp.
Published in **Int.J.Mod.Phys. E25 (2016) no.05, 1641007**
DOI: 10.1142/S021830131641007X
e-Print: **arXiv:1604.01332** **[nucl-th]** | PDF

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