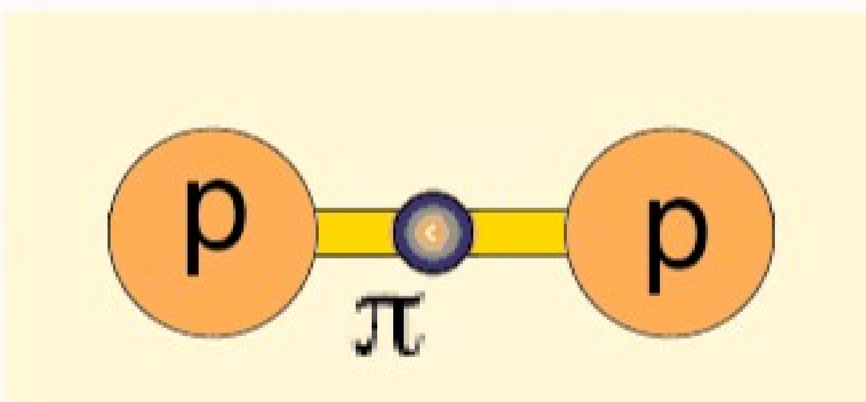
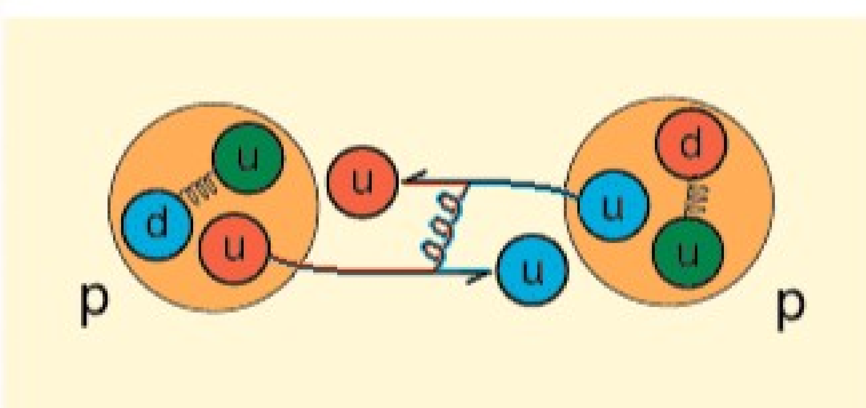
Nuclear Physics 6

Quantum Chromodynamics & Nuclear Physics

RECAP: nucleons composite

$\Downarrow$





nuclear forces are residual forces

↳ Ideally, we want to understand nuclear forces from $\underbrace{\text{QCD}}_{\text{What is this about?}}$

$\downarrow$

We begin w/ quantum electrodynamics (QED)

QED → theory of electrons & photons

↓

A QFT : we begin w/ a Lagrangian

$$\mathcal{L}_{QED} = \underbrace{\bar{\psi}}_{\text{electron field}} (i\not{D} - m)\psi - \frac{1}{4}\underbrace{F_{\mu\nu}F^{\mu\nu}}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

A_ν, A_μ ↓ photon field

electron field ↓ Dirac Field (spin-1/2 fermion)

photon field ↓ vector field (spin-1 boson)

For the moment the details are not crucial...

— ⊗ —

However we can understand QED as a gauge theore

1) Dirac field

$$\psi(x) \quad , \quad \mathcal{L} = \bar{\psi}(x)(i\not{\partial} - m)\psi(x)$$

$$\not{\partial} = \gamma_\mu \partial^\mu$$

→ Global U(1) Symmetry

$$[\psi(x) \to e^{ie\alpha(x)}\psi(x)]$$

↳ this type of transformation

Is this invariant under U(1)?

$$\psi(x) \to e^{iea}\psi(x)$$

$$\Downarrow$$

$$\bar{\psi}(x)\psi(x) \to \bar{\psi}(x)\psi(x)$$

$$(\bar{\psi}(x) = \psi^{\dagger}(x)\gamma^{0})$$

$$\bar{\psi}(x)\partial_{\mu}\psi(x) \to \bar{\psi}(x)\partial_{\mu}\psi(x)$$

$$\Downarrow$$

$$\mathcal{L} \to \mathcal{L}$$

$$\bar{\psi}(i\partial\!\!\!/ - m)\psi \to \bar{\psi}(i\partial\!\!\!/ - m)\psi$$

$\to$ Global U(1) Symmetry

But we also have local U(1):

$$\psi(x) \to e^{ie\underbrace{\alpha(x)}_{\text{phase depends on } x}}\psi(x)$$

$$\Downarrow$$

$$\bar\psi(x)\psi(x) \to \bar\psi(x)\psi(x) \quad ✓$$

$$\bar\psi(x)\partial_\mu\psi(x) \to \bar\psi(x)\partial_\mu\psi(x) \quad ✗$$

$$\underbrace{+\, ie\,\partial_\mu\alpha\,\bar\psi(x)\psi(x)}_{\text{this one fails!}}$$

$$\mathcal{L} \not\to \mathcal{L}$$

But there's a solution:

1) Introduce a new field.

$$A_\mu \to A_\mu + \partial_\mu \alpha$$

U(1) transformation rule

2) Define a covariant derivative

$$\cancel{\partial_\mu} \Rightarrow D_\mu = \partial_\mu - ieA_\mu$$

And now magic happens:

$$\bar{\psi}\psi \to \bar{\psi}\psi$$

$$\bar{\psi} D_\mu \psi \to \bar{\psi} D_\mu \psi \quad \text{(check it)}$$

Thus we have to write:

$$\mathcal{L} = \bar{\psi}(i\cancel{D} - m)\psi + \underbrace{(\dots)}_{\text{piece containing } A^\mu}$$

covariant derivative

Let's find it

$$A_\mu \to A_\mu + \partial_\mu \alpha$$

$$\Downarrow$$

$$\partial_\nu A_\mu - \partial_\mu A_\nu \to \partial_\nu A_\mu - \partial_\mu A_\nu$$

(Invariant!)

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Now we build the $F_{\mu\nu}$ piece

$$\Rightarrow \mathcal{L} = \bar{\psi}(i\not{D} - m)\psi + \lambda F_{\mu\nu}F^{\mu\nu}$$

$$\Downarrow$$

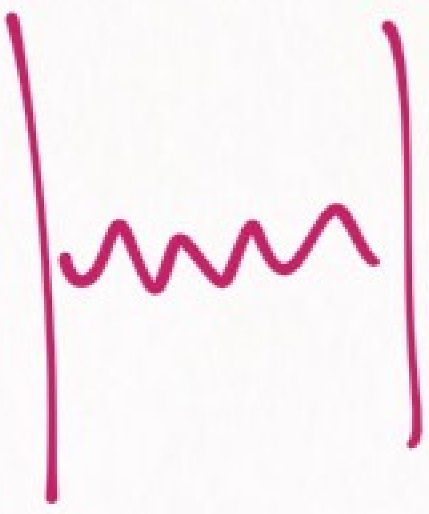
We can fix λ from the condition of having a kinetic term like:

$$\mathcal{L} = \frac{1}{2} A_\mu \Box^2 A^\mu + \ldots$$

$$\Downarrow$$

$$\boxed{\lambda = -1/4}$$ (Exercise)

$$\Rightarrow \mathcal{L}_{QED} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$\mathcal{L}_{QED} \Rightarrow$  $= \frac{\alpha}{r}$ (Coulomb)

What about QCD?

$\Rightarrow$ Just like QED, but w/ more charges

QED: 1 charge (electric)

QCD: 3 charges (color)

RGB

Local symmetry:

$$U(1) \longrightarrow \underbrace{SU(3)}$$

3×3 matrices U / $U^\dagger U = 1$

$\det U = 1$

QED & QCD

→ 3 types of charge
(color)

→ 1 type of charge
(electric charge)

QED → photons & electrons

QCD → gluons & quarks

6 types

[u, d, s, c, b, t]

$$\mathcal{L}_{QCD} = \sum_{i=1}^{n_F} \bar{q}_i (i\not{D} - m_i) q_i - \frac{1}{4} \underbrace{G^{a}_{\mu\nu} G^{\mu\nu,a}}_{\text{gluon fields}}$$

$\bar{q}_i$ → quark fields

$$D^{a}_{\mu} = \partial_\mu - ig \sum_a \frac{\lambda^a}{2} A^{a}_{\mu}$$

$$F^{a}_{\mu\nu} = \partial_\mu A^{a}_{\nu} - \partial_\nu A^{a}_{\mu} + g f_{abc} A^{b}_{\mu} A^{c}_{\nu}$$

$$q_i = \{u, d, s, c, b, t\}$$

$$\left[\frac{\lambda^b}{2}, \frac{\lambda^c}{2}\right] = i \underbrace{f^{abc} \frac{\lambda^a}{2}}_{\text{non-abelian group}} \rightarrow \text{SU(3) generators}$$

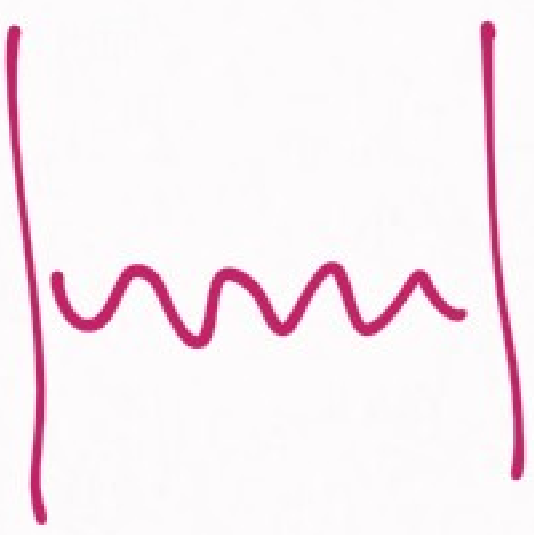
A complicated version of QED

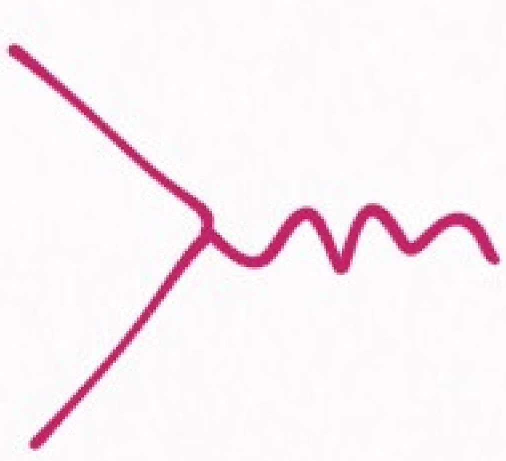
But the most important difference is not $\underbrace{N_c}_{\#\text{ colors}}$ or $\underbrace{N_F}_{\#\text{ flavors}}$

Let's meet asymptotic freedom

→ but first we go back to

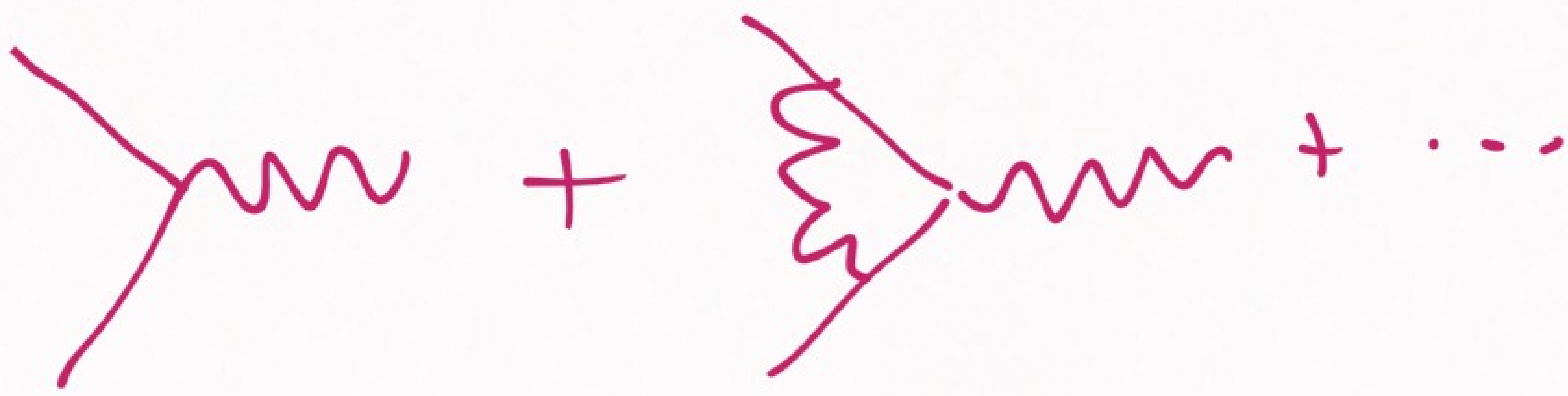
Coulomb

 $= 4\pi \dfrac{\alpha}{|q|^2}$

 $\propto \sqrt{\alpha}$

the electron vertex function

QFT $\rightarrow$ quantum corrections



$=$ 

$\propto \sqrt{\alpha(q^2)}$

$\Rightarrow$ the coupling $\alpha = \alpha(q^2)$

(depends on photon energy)

$$\Rightarrow \alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log\left(\frac{q^2}{\mu^2}\right)}$$

Relates α for q^2 & μ^2

Now, look carefully:

$$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log\left(\frac{q^2}{\mu^2}\right)}$$

$$\Rightarrow \alpha(q^2 > \mu^2) > \alpha(\mu^2)$$

→ Strength of e.m. interaction grows w/ energy

$$\alpha(m_e^2) \simeq \frac{1}{137} \Rightarrow \alpha(\Lambda_0^2) \to \infty$$

$$\Rightarrow \boxed{\Lambda_0 = m_e \exp\left(\frac{3\pi}{2\alpha}\right) \simeq 10^{280}\ \mathrm{MeV}}$$

THE LANDAU POLE

Landau Pole $\rightarrow$ energy at which QED breaks down

$\downarrow$

$\Lambda_0 \gg M_{pl}$

$\hookrightarrow$ Mostly a theoretical problem

QCD → similar effect

+ ... → more complex to calculate

WHY?

gluons carry color

(photons were neutral)

→ $\alpha_s(q^2)$ will be different than $\alpha(q^2)$

$$\alpha_s(q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi}(33-2n_f)\log\frac{q^2}{\mu^2}}$$

$$\alpha_s(q^2 > \mu^2) < \alpha_s(\mu^2)$$

↳ QCD becomes weaker at short-distances

$$\Lambda_{QCD} = \mu \exp\left[-\frac{12\pi}{(33-2n_f)\alpha_s(\mu^2)}\right]$$

↳ $\boxed{\alpha_s(\Lambda_{QCD}^2) \to \infty}$

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2n_f)\log\frac{Q^2}{\mu^2}}$$

QCD → Λ_{QCD} → natural QCD scale

Why?

$\underbrace{m_u, m_d, m_s}_{\text{light quarks}} \ll \Lambda_{QCD} \ll \underbrace{m_c, m_b, m_t}_{\text{heavy quarks}}$

ρ mass $(u\bar{d}) \rightarrow 770$ MeV

$\simeq 2\Lambda_{QCD}$

p mass $(uud) \rightarrow 940$ MeV

$\simeq 3\Lambda_{QCD}$

$\Lambda_{QCD} \gg m_u, m_d$, that's why!

D meson $(c\bar{u})$ mass

$\rightarrow m_c + \Lambda_{QCD}$

$\underbrace{(1.5 + 0.3)\,\text{GeV}}$ $\simeq 1.8\,\text{GeV}$

But depends on choice of m_c

$\rightarrow m_c + 2\Lambda_{QCD}$ also works

$(1.2 + 0.6)\,\text{GeV} \simeq 1.8\,\text{GeV}$

(quark masses non-observable)

$\Big[\, m_c \simeq 1.5\,\text{GeV}$ includes $\Lambda_{QCD} \,\Big]$

— ⊗ —

$\eta, J/\psi$ $(c\bar{c})$ masses

$\rightarrow 2m_c + 2\Lambda_c$

$(2.4 + 0.6)\,\text{GeV} \simeq 3\,\text{GeV}$

Λ_{QCD} natural QCD scale

↳ what happens w/ pion?

π mass ($u\bar{d}$):

Expectation → $2\Lambda_{QCD}$

(0.6 - 0.7 GeV)

Reality → 0.14 GeV !

$$\boxed{\frac{m_\pi}{2\Lambda_{QCD}} \sim \frac{1}{6} - \frac{1}{5}}$$

⇓

FINE TUNING

⇓

WHY? → SYMMETRY

QCD & nuclear forces

1) pion mass unnatural

$\longrightarrow$ [CHIRAL SYMMETRY]

—— $\otimes$ ——

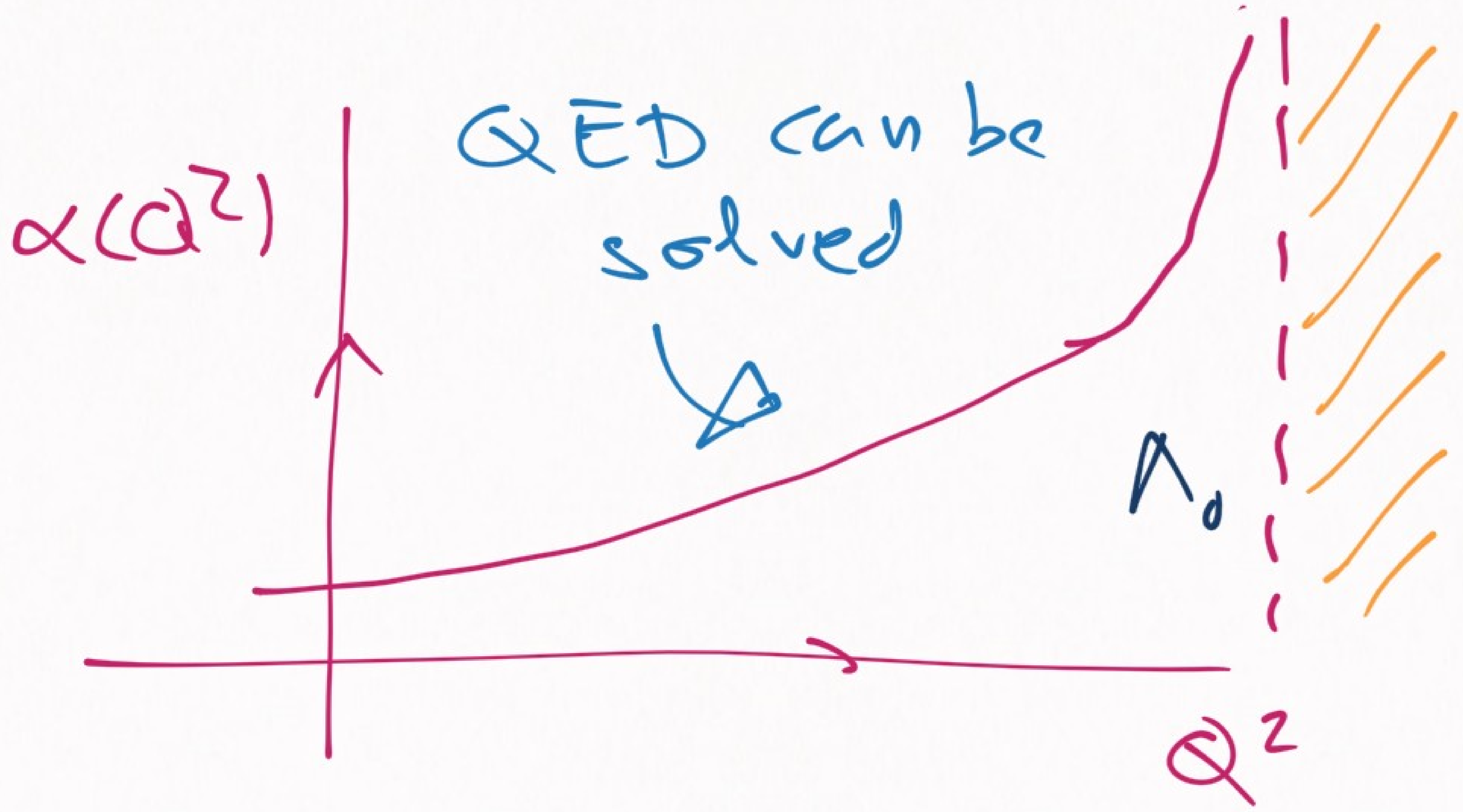
But there is a second issue

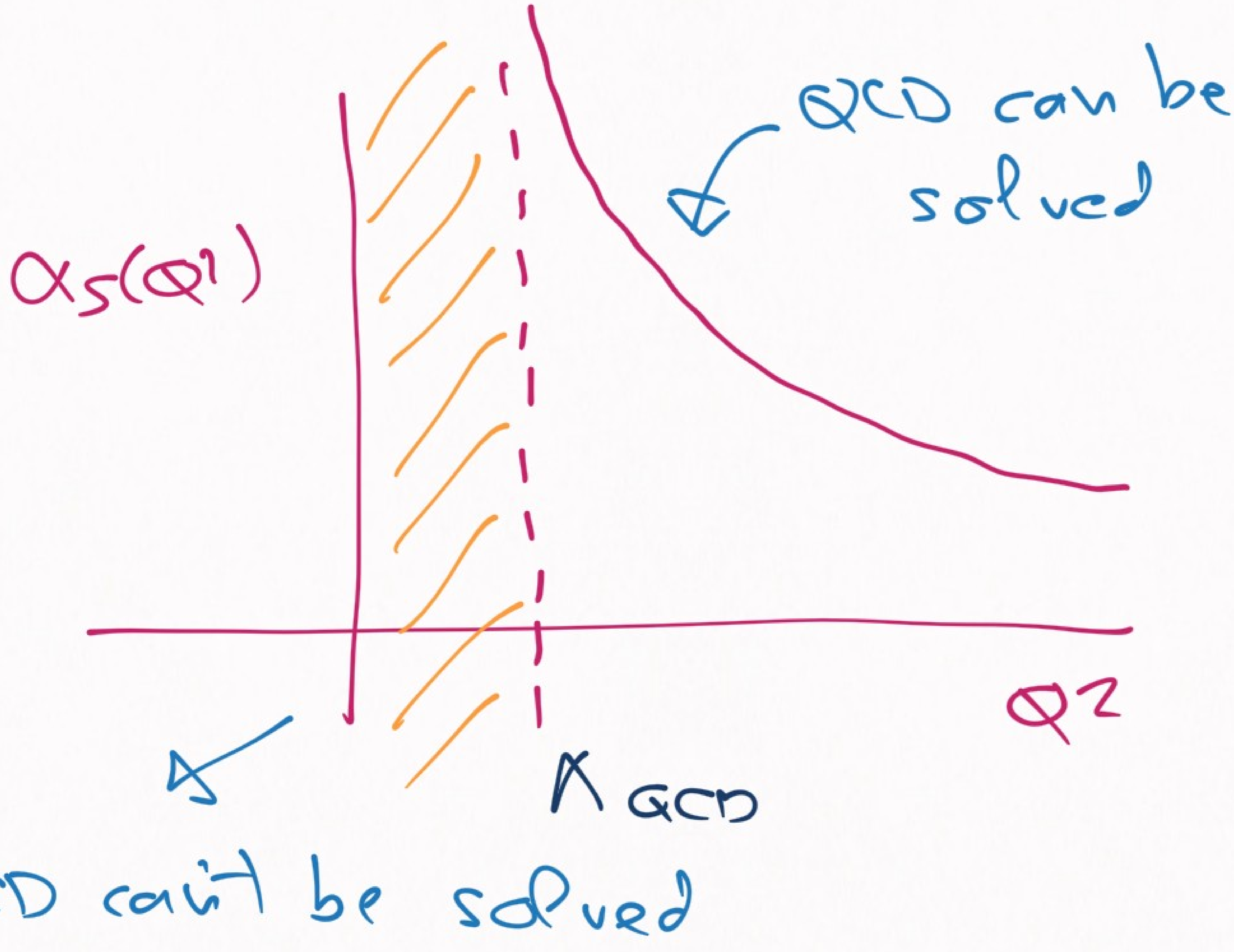
↳ look at $\alpha(Q^2)$ & $\alpha_s(Q^2)$

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log\left(\frac{Q^2}{\mu^2}\right)}$$

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2n_f)\log\left(\frac{Q^2}{\Lambda^2_{QCD}}\right)}$$

GRAPHICAL REPRESENTATION:



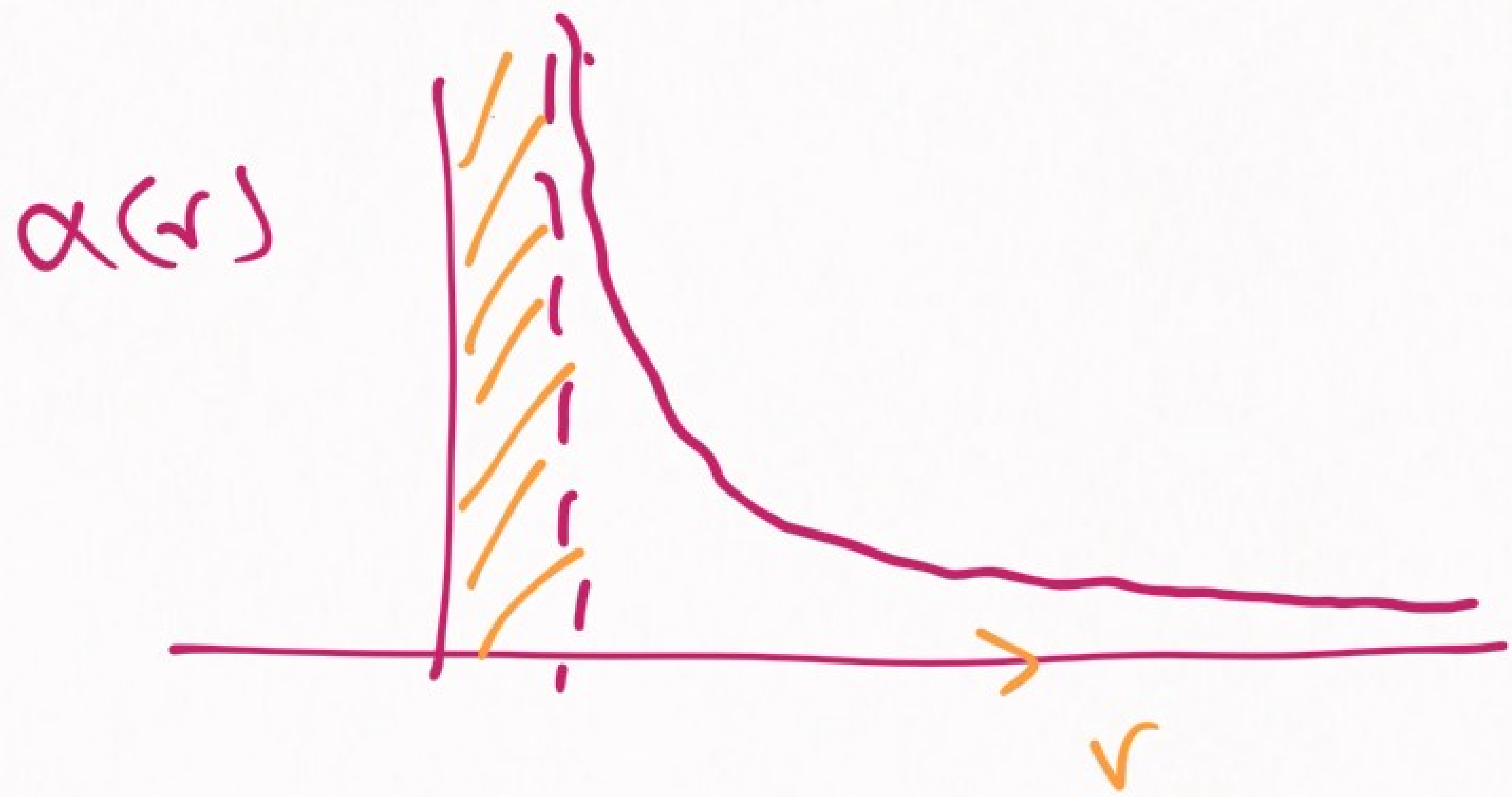


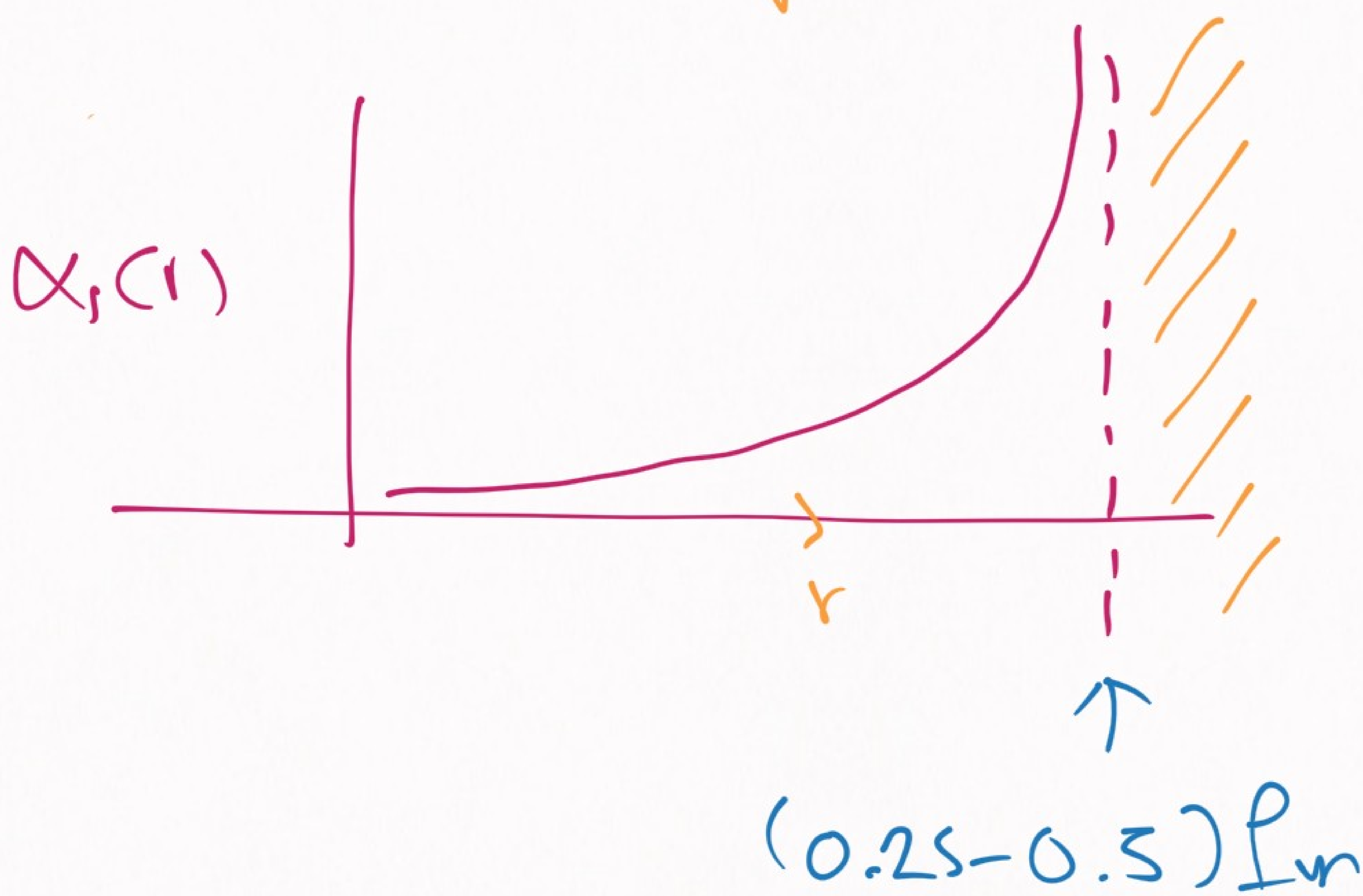
$$\Lambda_0 \simeq 10^{280}\ \text{GeV}$$

$$\Lambda_{QCD} \simeq 0.2 - 0.3\ \text{GeV}$$

↰ that was energy

& this is distances ↴





→ $\boxed{\text{QCD can't be solved at long distances}}$

[QCD cannot be solved at long distances (> 0.5 fm)]

$\Downarrow$

[Proton size about 0.5 - 1.0 fm]

$\Downarrow$

[ERGO... We cannot directly derive nuclear physics from QCD]

$\Downarrow$

OUPS !

FUNDAMENTAL PROBLEM OF NUCLEAR PHYSICS

$\boxed{\text{How do we derive nuclear forces from QCD?}}$

$\downarrow$

Bad news, we can't do it (directly)

$\downarrow$

NEXT LECTURE:

[The solutions to this deadlock]