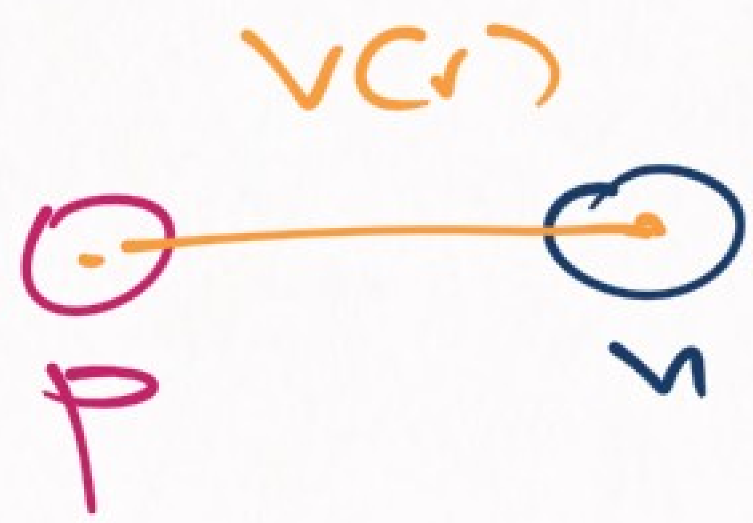
# Nuclear Physics (4)

A general overview

of nuclear forces

# Properties of the Nuclear Force




How is $V(r)$?

1) Short-ranged
2) Attractive at intermediate distances
3) Repulsive at short distances
4) Does not distinguish nectrons & protons
5) Not central

1) Short-ranged

Two types of forces

1.a) Long-ranged : power-law damping

$$V(r) \sim \frac{1}{r^n}$$

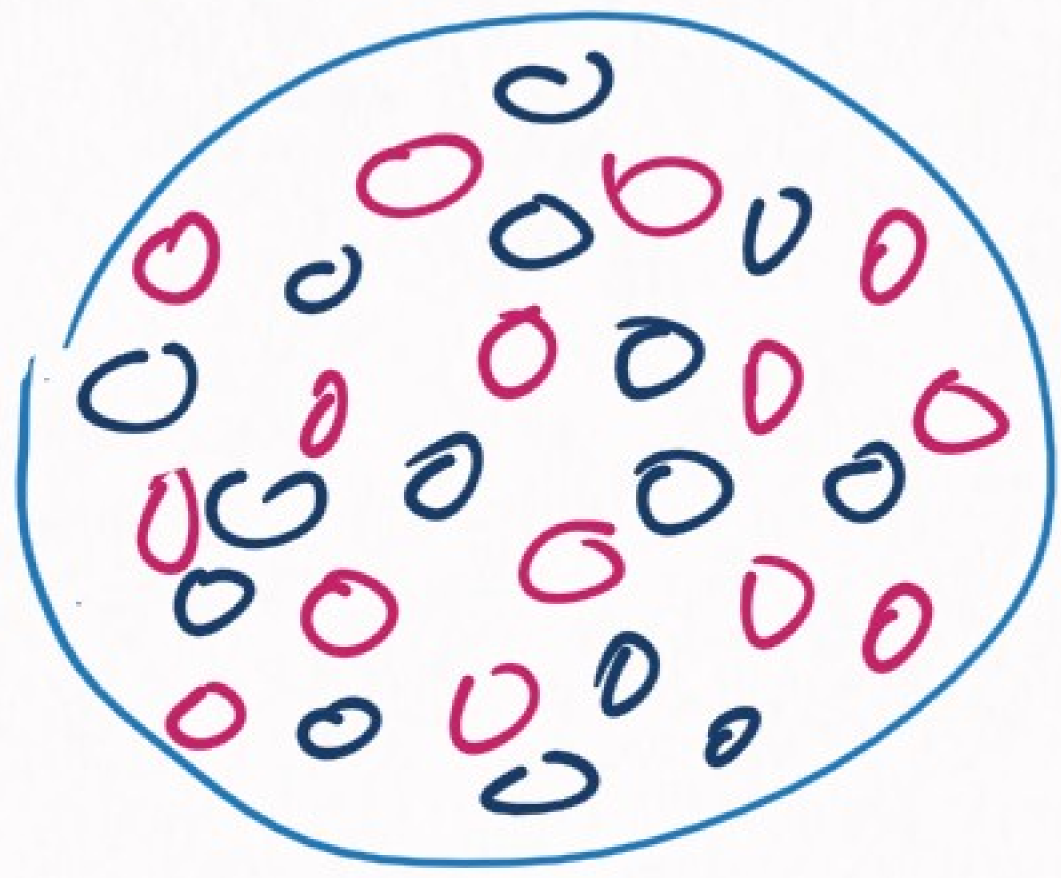
→ Coulomb, gravity, van der Waals

1.b) Short-ranged : exponential damping

$$V(r) \sim \frac{e^{-\lambda r}}{r^n}$$

→ Yukawa

1) Cont'd | Why nuclear force is short-range?



$$\boxed{\frac{B}{A} \sim 8\,\text{MeV}}$$

SATURATION

B -> binding energy

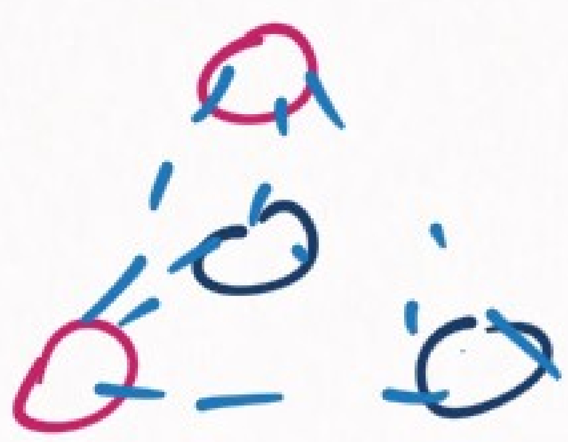
A -> # of nucleons

Long-ranged : $\frac{B}{A^2} \sim \text{const}$ (1)

Short-ranged : $\frac{B}{A} \sim \text{const}$ (2)

(1) prop to number of interactions

(2) prop to number of particles



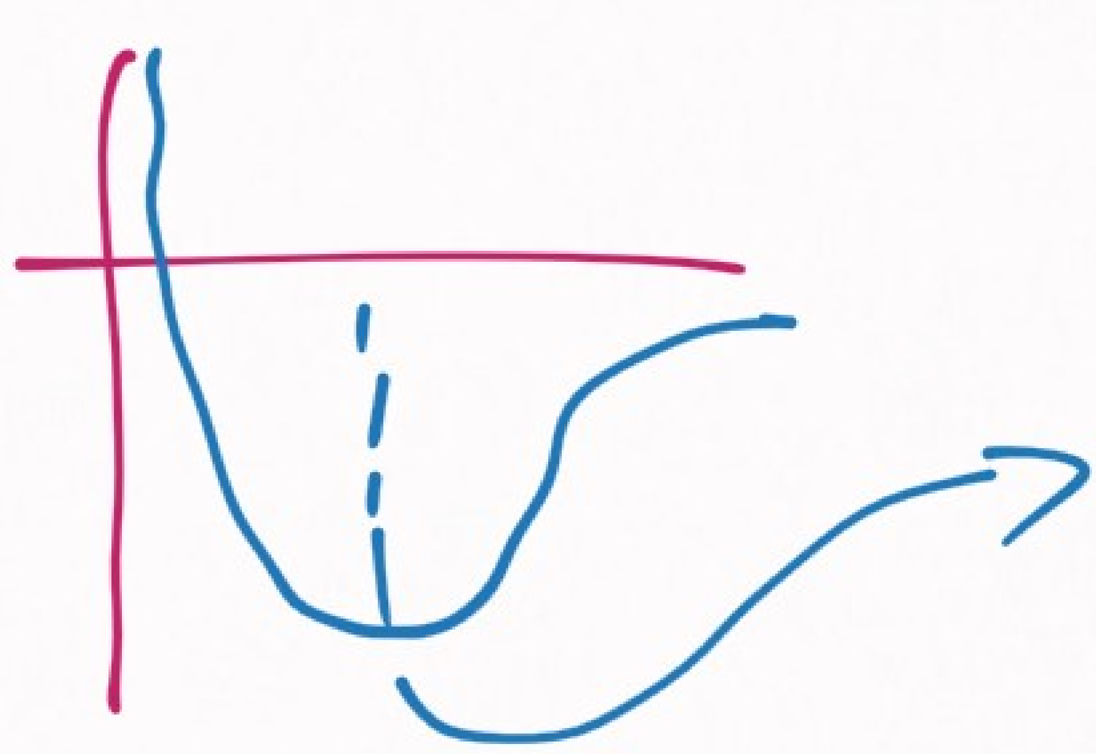
$\alpha$-particle:

range $\sim 1.7$ fm

(Wigner's estimation)

(modern estimation $\sim 1.4$ fm)

2) Attractive at medium distances



a minimum?

Nucleon density of heavy nuclei

$\sim 0.17$ nucleons/fm$^3$

$\sim \left(\frac{1}{R_{min}}\right)^3 \Rightarrow R_{min} \sim 1.8$ fm

3) Repulsive core

$^1S_0$ phase shift


the zero suggests a repulsive core

$$R_{core} \sim \frac{1}{K_{zero}} \sim 0.6 - 0.7 \text{ fm}$$

$K_{zero}$ → 300 MeV

4) Charge independence :

neutrons ⟷ protons

$B(^3He) \sim B(^3H)$

$^3He$ $\quad$ $^3H$

7.72 MeV $\quad$ 8.48 MeV

coulomb repulsion

5) Non-central

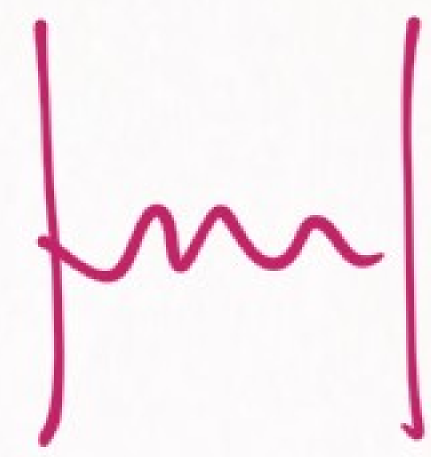
→ The deuteron is not round

$V \neq V(r)$ but more complex

→ we check this later

What is the origin of nuclear forces?

QFT → Forces come from exchanging bosons

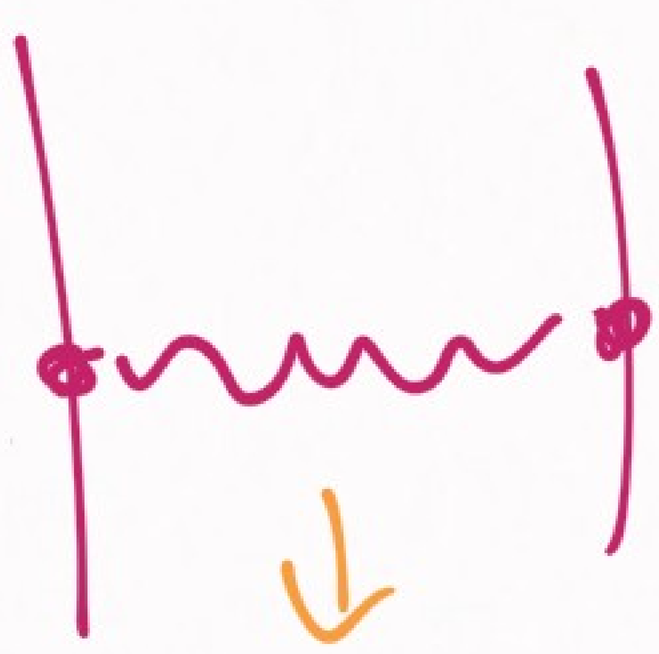
Coulomb:  $\sim \dfrac{e^2}{|\vec{q}|^2}$

(exchange of a photon)

$\longrightarrow$ can be calculated from Feynman rules

Peskin & Schröder

Zee



$= \dfrac{e^2}{|\vec{q}|^2}$

$e \times 1/|\vec{q}|^2 \times e$

$\longrightarrow$ then we Fourier-transform

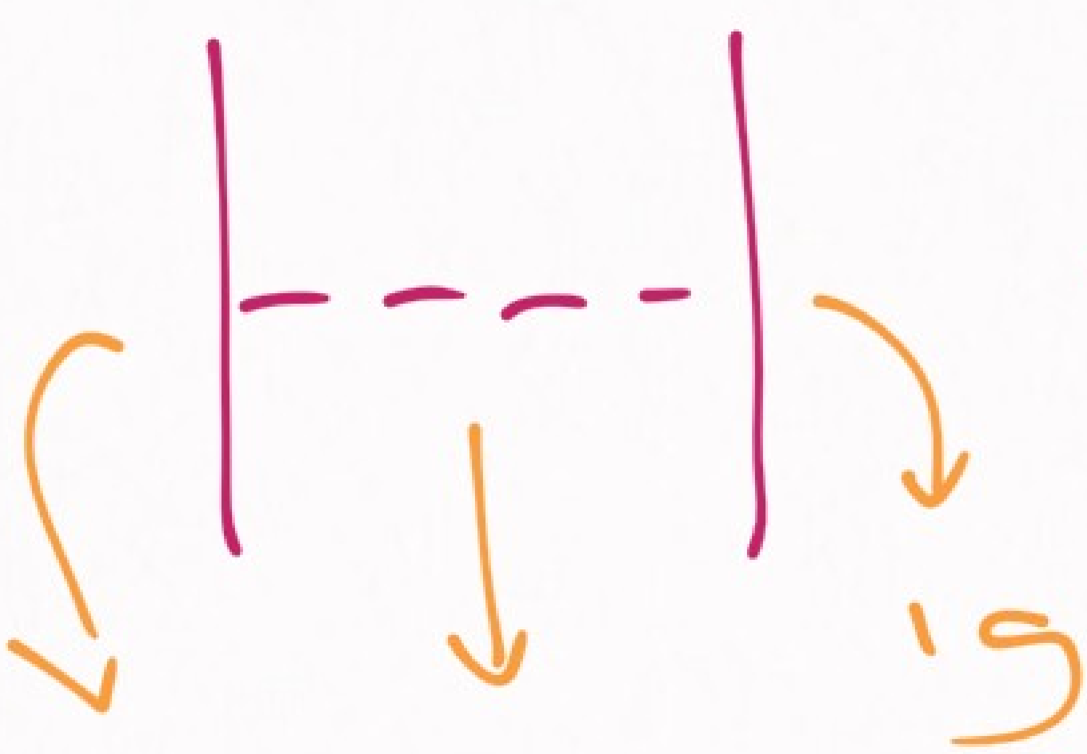
$$V(\vec{r}) = e^2 \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{e^{i\vec{q}\cdot\vec{r}}}{|q|^2} = \frac{e^2}{4\pi r} = \frac{\alpha}{r}$$

w/ $\dfrac{e^2}{4\pi} = \alpha \simeq \dfrac{1}{137}$

Then we can try this w/ nucleons:

Yukawa: a massive boson
(the pion)

Let's try $J^P = 0^+$ (scalar)



$$\sim -\frac{g^2}{\vec{q}^2 + m^2}$$

$ig$ $\quad 1/m^2 + |\vec{q}|^2 \quad$ $ig$

$$V(\vec{r}) = -\int \frac{d^3\vec{q}}{(2\pi)^3}\, e^{-i\vec{q}\cdot\vec{r}} \cdot \frac{g^2}{\vec{q}^2 + m^2} = -g^2 \frac{e^{-mr}}{4\pi r}$$

The Yukawa potential!

Yukawa: $m \sim (100-200)$ MeV

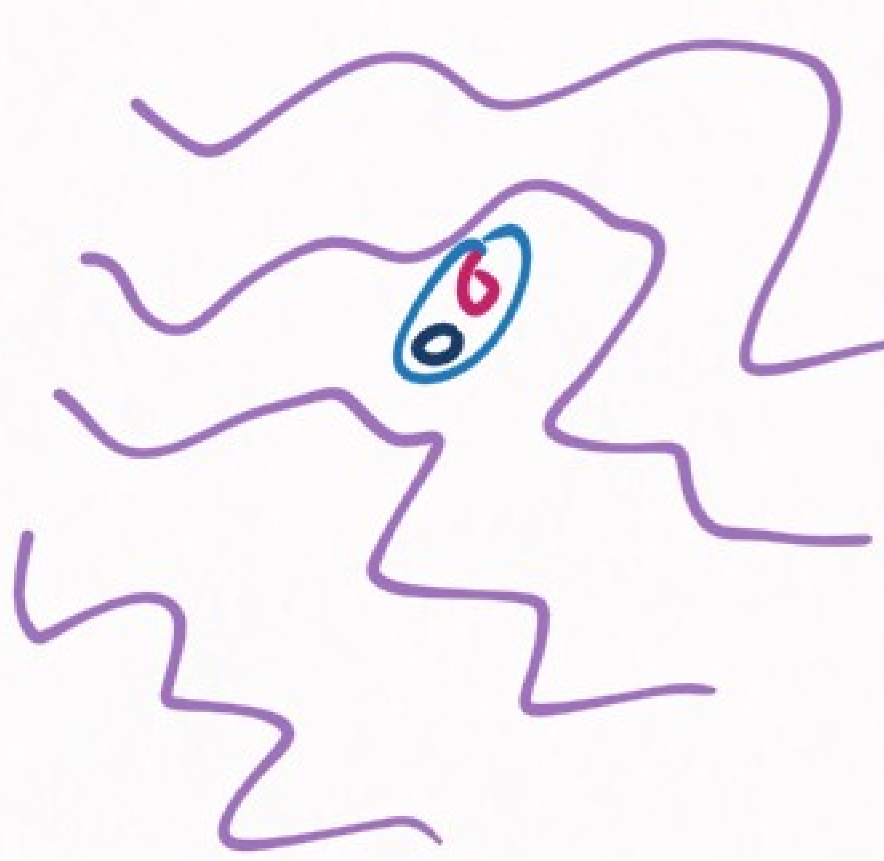
1935

Problem: $V(r) = -g^2 \frac{e^{-mr}}{4\pi r}$

is central...

but deuteron has a quadrupole moment (i.e. not spherical)

Deuteron in electric field



$\Phi(\vec{r})$

↳ electric potential

$$V_{det} = q\Phi + \vec{d}\cdot\vec{\nabla}\Phi + \frac{1}{6}Q_{ij}\partial_i\partial_j\Phi + \dots$$

charge

dipole moment

quadrupole moment

$$q = \int d^3\vec{r}\, \rho(\vec{r})$$

Charge distribution

$$\vec{d} = \int d^3\vec{r}\, \rho(\vec{r})\, \vec{r}$$

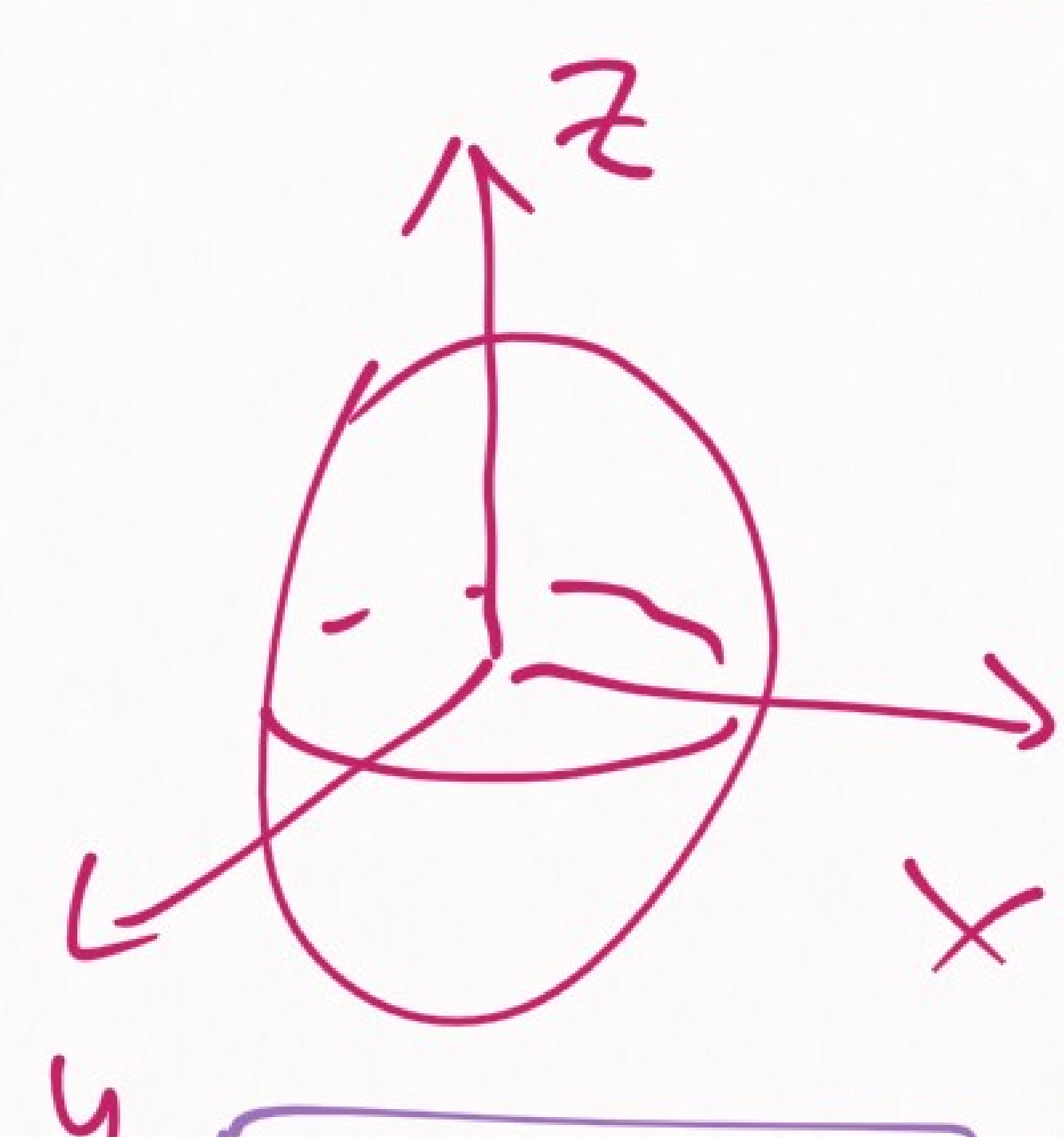
$$Q_{ij} = \int d^3\vec{r}\, \rho(\vec{r})\left(3 r_i r_j - r^2 \delta_{ij}\right)$$

$Q = Q_{33} \rightarrow$ quadrupolar moment

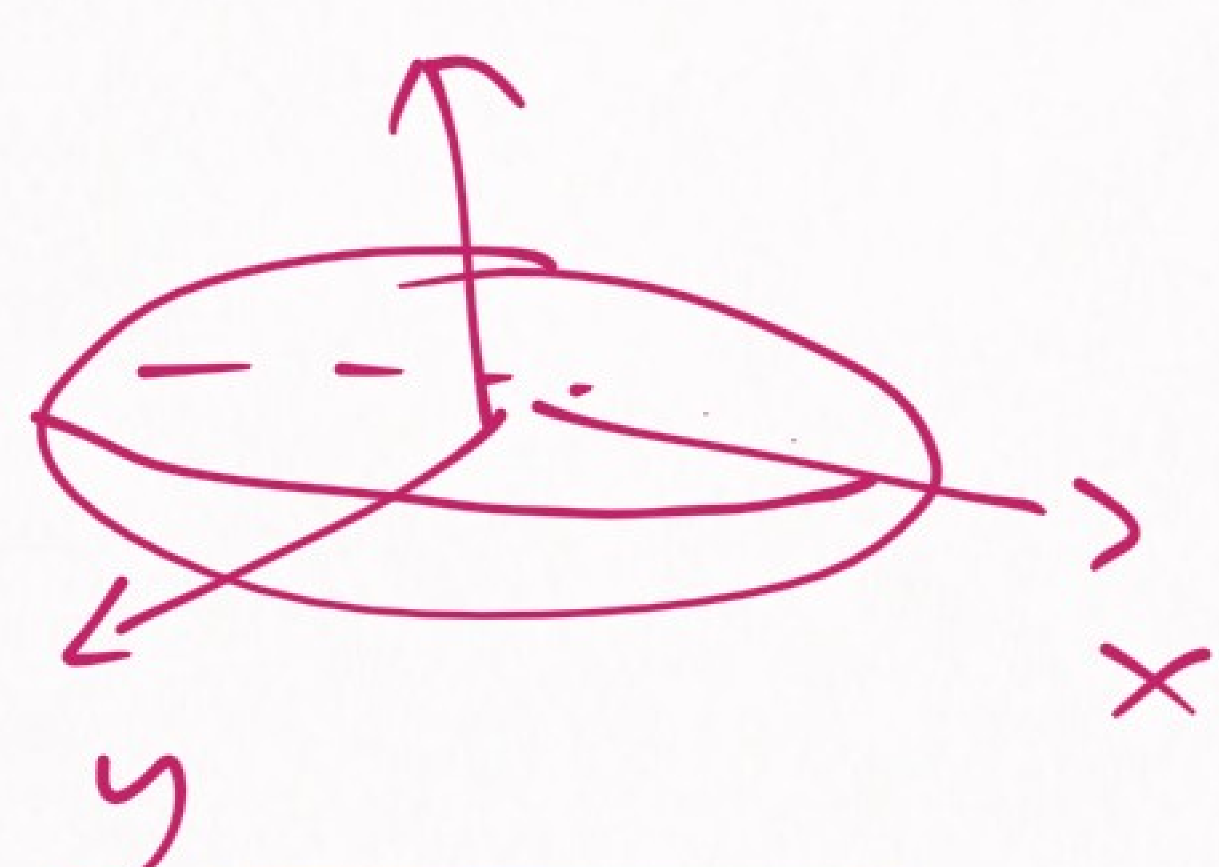
 $\longrightarrow$ $Q = 0.286\ \text{fm}^2$

deuteron

$$Q = \int d^3\vec{r}\, \rho(\vec{r})\left(3z^2 - r^2\right)$$



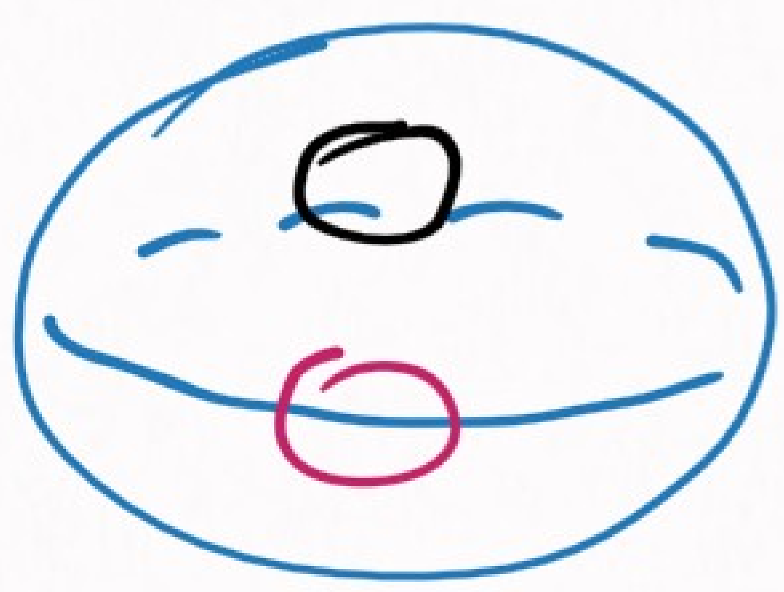

$\boxed{Q > 0}$




$\boxed{Q < 0}$

Problem: $V(\vec{r}) = -g^2 \frac{e^{-mr}}{4\pi r}$

gives us a spherical deuteron

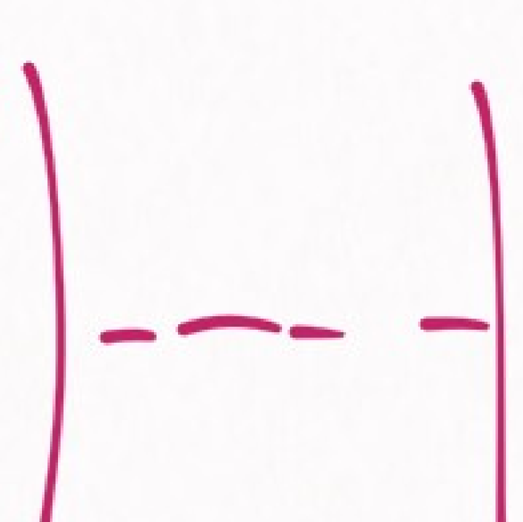


$\rightarrow$ but it's not spherical

$$\left[\text{Conclusion: the pion cannot be a } J^P = 0^+ \text{ boson}\right]$$

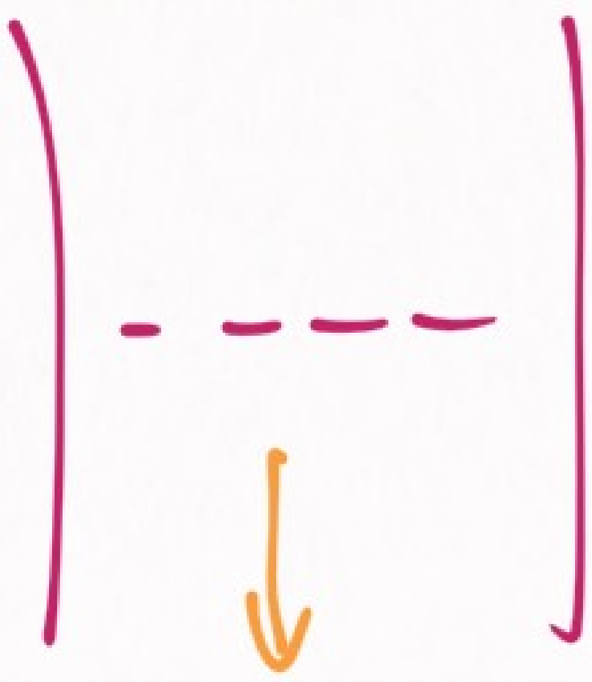
Well, let's try another $J^P$

$\boxed{J^P = 1^-}$ $\rightarrow$ vector boson (like a heavy photon)



$\rightarrow$ two pieces: electric & magnetic

## Electric piece



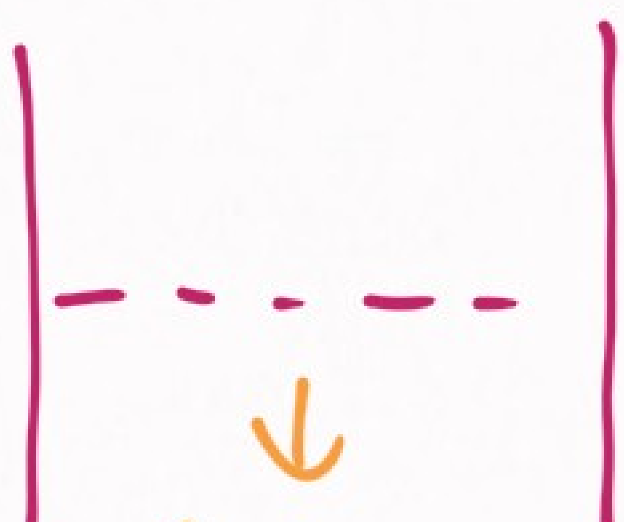
$g$

$\dfrac{1}{\vec{q}^2+m^2}$

$g$

$$\sim \underbrace{\frac{g^2}{\vec{q}^2+m^2}}_{\text{a repulsive Yukawa}}$$

## Magnetic piece




$$\sim \frac{f^2}{2M}\,\frac{(\vec{\sigma}_1\cdot\vec{q})\cdot(\vec{\sigma}_2\cdot\vec{q})}{\vec{q}^2+m^2}$$

$1/\vec{q}^2+m^2$

$i\dfrac{f}{2M}(\vec{\sigma}_2\cdot(-\vec{q}))$

$i\dfrac{f}{2M}(\vec{\sigma}_1\cdot\vec{q})$

$\vec{\sigma}_i = \dfrac{\vec{S}_i}{2}$ ($\vec{S}_i$ the spin of nucleon $i=1,2$)

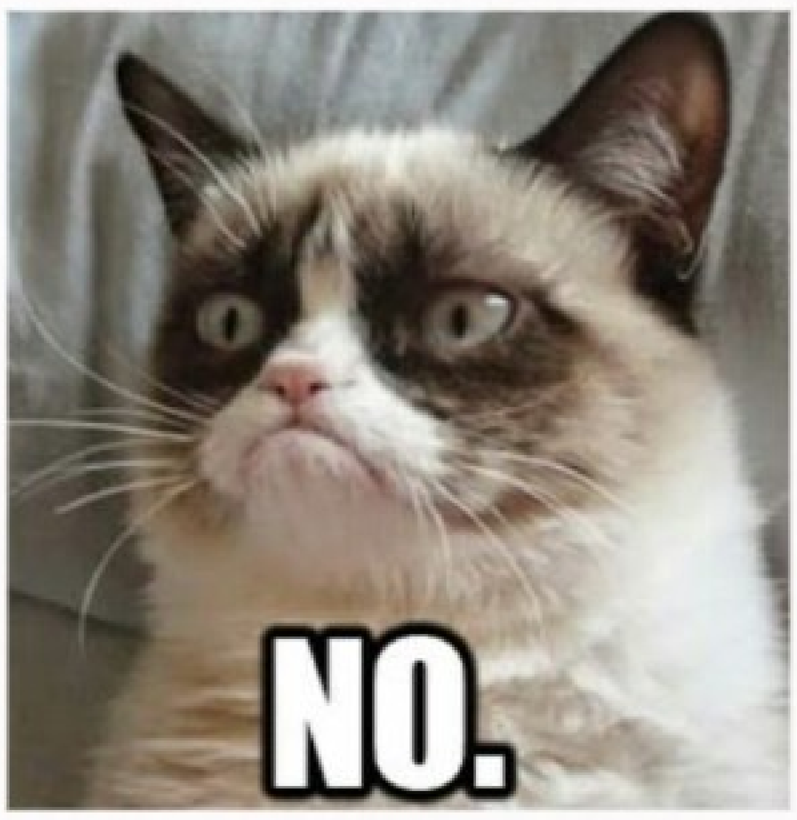
The interesting piece is the magnetic one

$$V_M(\vec{r}) = \left(\frac{f}{2M}\right)^2 \int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \frac{(\vec{\sigma}_1 \wedge \vec{q})\cdot(\vec{\sigma}_2 \wedge \vec{q})}{\vec{q}^2 + m^2}$$

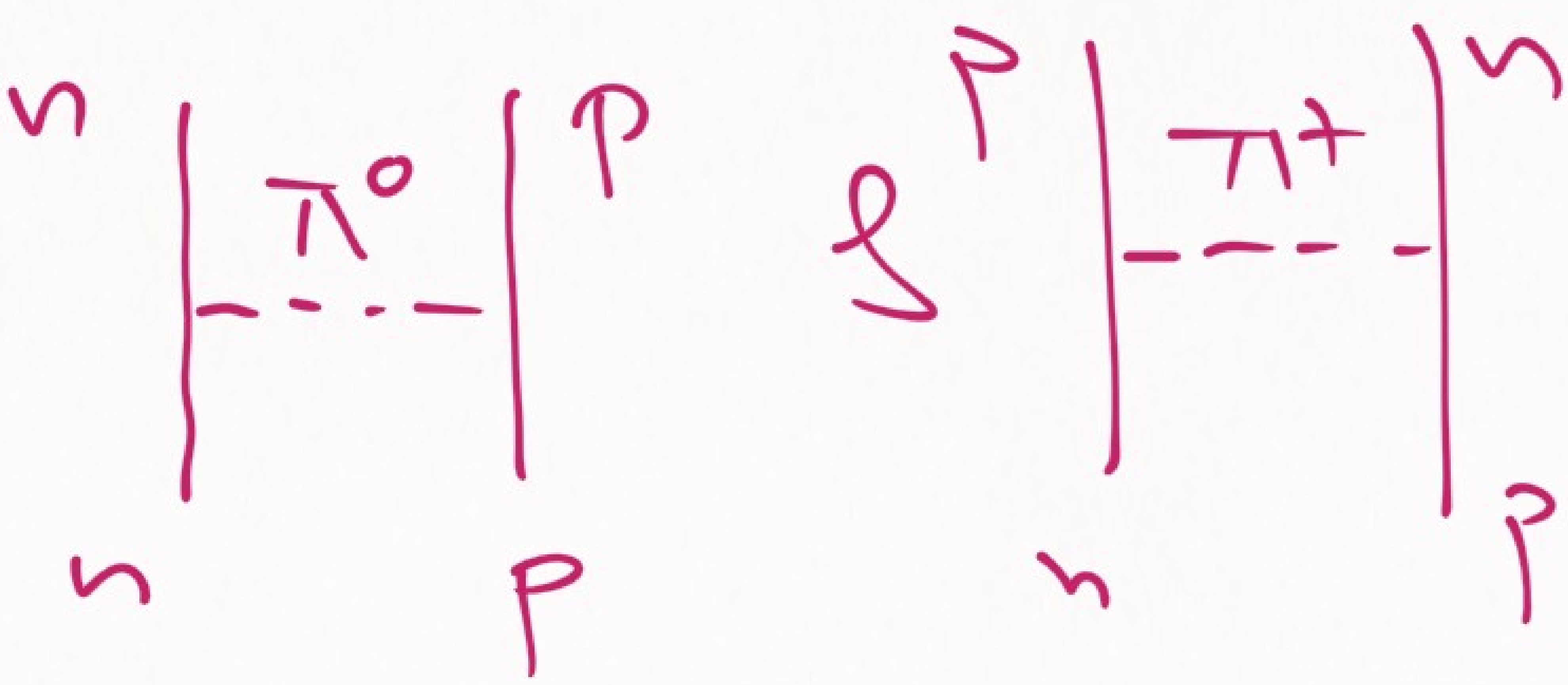
$$= \left(\frac{f}{2M}\right)^2 \Big[ \frac{2}{3} \vec{\sigma}_1 \cdot \vec{\sigma}_2 \frac{e^{-mr}}{4\pi r}$$

$$- \frac{1}{3} \underbrace{(3\, \vec{\sigma}_1 \cdot \vec{r}\; \vec{\sigma}_2 \cdot \vec{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2)}_{\text{Does not look spherical}} \frac{e^{-mr}}{4\pi r} \underbrace{(\cdots)}_{\text{something complicated}} \Big]$$

↳ Did we solve it?



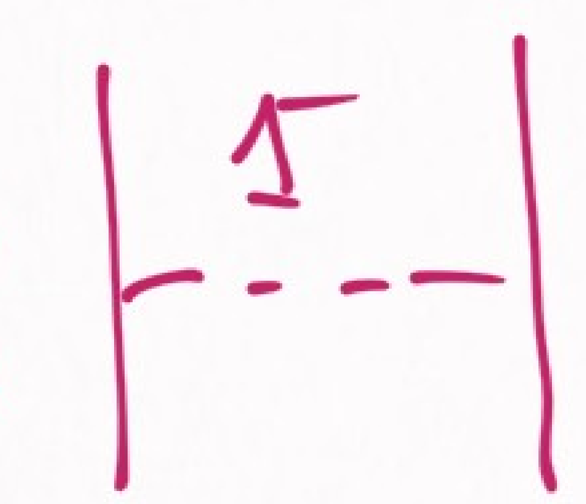
Deuteron : neutron-proton potential




→ a complication

Can be solved w/ a factor:

$$V_d(\vec{r}) = -3 \times \Big($$




$$\Big)$$

$$= -3\left[ V_E(\vec{r}) + V_M(\vec{r}) \right]$$

$$V_E(\vec{r}) = g^2 \frac{e^{-mr}}{4\pi r} \qquad V_M(\vec{r}) = \text{(see previous page)}$$

Problem → this give $Q < 0$

(try it as an exercise)

$\cancel{0^+}$ $\cancel{1^-}$ Let's try $0^-$ !

(pseudoscalar)


$$\sim -\left(\frac{g}{2M}\right)^2 \frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{q^2+m^2}$$

$1/(\vec{q}^2+m^2)$

$\frac{g}{2M}\vec{\sigma}_1\cdot\vec{q}$ $\qquad$ $\frac{g}{2M}\vec{\sigma}_2\cdot(-\vec{q}) \leftarrow \gamma^5$

Now we Fourier Transform:

$$V_{P}(r) = -\left(\frac{g}{2M}\right)^2 \int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{q^2+m^2}$$

$$= \left(\frac{g}{2M}\right)^2 \Big[ \vec{\sigma}_1\cdot\vec{\sigma}_2 \frac{e^{-mr}}{4\pi r}$$

$$+ (3\vec{\sigma}_1\cdot\hat{r}\,\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_1) \frac{e^{-mr}}{4\pi r}(\dots) \Big]$$

Now we add the mysterious
(−3) factor

$$V_d(\vec{r}) = -3 V_p(\vec{r})$$

And finally $\boxed{Q > 0}$ (exercise)

—— ⊗ ——

We got it! The pion is $0^-$
(a pseudoscalar boson)

## What have we learned?

→ properties of nuclear force

- short-ranged
- non-central ($Q_d > 0$)

→ Yukawa's idea:

exchange of a boson

- massive boson $m > 0$

  ⇒ explains short-ranged

- different types of boson

→ $0^+$ ⇒ central force

→ $1^-$ ⇒ non-central, $Q_d < 0$

→ $0^-$ ⇒ non-central, $Q_d > 0$

↳ fits the bill!