Nuclear Physics 2

Why nuclear physics
is so damn
difficult

(while atomic physics
is so easy)

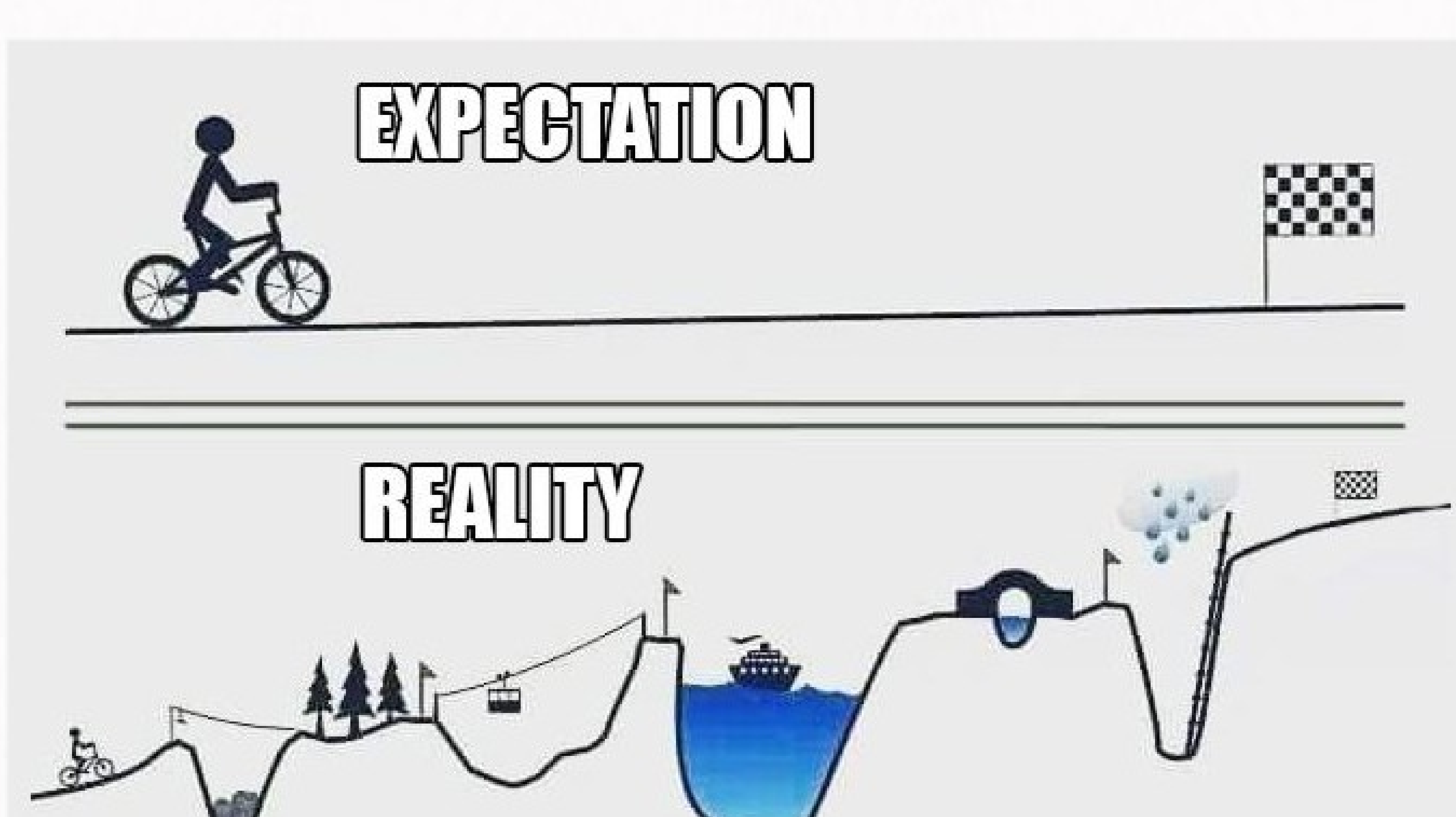
Back to the hydrogen atom:

$$\left.\begin{aligned} r_B &= \frac{c_B}{a_B} \\ \sqrt{\langle r^2 \rangle} &= \frac{d_B}{a_B} \end{aligned}\right\} \circledast$$

$\circledast \rightarrow$ Expectation:

$$c_B \sim O(1)$$
$$d_B \sim O(1)$$

But we know how things go...



But this time, for once, expectations match reality!

$$\left[-\nabla^2 - \frac{2}{a_B r}\right]\psi(\vec{r}) = -\gamma_B^2\,\psi(\vec{r})$$

$$\Rightarrow \psi(\vec{r}) = \frac{1}{\sqrt{4\pi}}\,\frac{2}{a_B^{3/2}}\,e^{-r/a_B}$$

$$\text{w/ } \gamma_B = \frac{1}{a_B}\,!$$

$$E_B = -\frac{1}{2\mu}\left(\frac{1}{a_B}\right)^2 = -\frac{1}{2\mu}(\mu\alpha)^2 = (*)$$

$$(*) = -\frac{1}{2}\mu\alpha^2 \simeq -m_e\,\frac{1}{2}\left(\frac{1}{137}\right)^2$$

And w/ the wave function we can compute $\sqrt{\langle r^2 \rangle}$

$$\langle r^2 \rangle = \int d^3\vec{r}\, |\psi(\vec{r})|^2 r^2$$

$$= \int_0^\infty r^2 dr \frac{4}{a_B^3} e^{-2r/a_B} r^2$$

$$= \frac{a_B^2}{8} \underbrace{\int_0^\infty dx\, x^4 e^{-x}}_{24} =$$

$$= 3a_B^2$$

$$\Rightarrow \boxed{\sqrt{\langle r^2 \rangle} = \sqrt{3}\, a_B}$$

To summarize:

$$\left[r_B = \frac{1}{a_B} \, , \sqrt{\langle r^2 \rangle} = \sqrt{3}\, a_B \right]$$

$$\left. \begin{aligned} c_B &= 1 \\ d_B &= \sqrt{3} \end{aligned} \right\} \text{ expectations were met!}$$

When this happens
we call this
"a natural problem"

Natural problem:

(i) there is a characteristic scale

Example: hydrogen atom

→ mass of electron

× strength of Coulomb

$$Q_B = m_e \alpha \simeq 3.7\,\mathrm{keV}$$

$$a_B = \frac{1}{Q_B} \simeq 0.54\,\mathring{A}$$

(ii) everything else is $\mathcal{O}(1)$ in terms of the characteristic scale

Example: hydrogen atom

$r_B \sim Q_B \qquad E_B \sim -\frac{1}{2m_e} Q_B^2$

$\sqrt{\langle r^2 \rangle} \sim a_B$

What is the speed of an electron inside a hydrogen atom?

Well... $\langle p \rangle \sim Q_B$

$p = m_e v \Rightarrow \langle v \rangle \sim \frac{Q_B}{m_e}$

$\Rightarrow \langle v \rangle \sim \alpha \sim \frac{c}{137}$

$\sim 2200\ \mathrm{km/s}$!

Moral: natural problems are easy

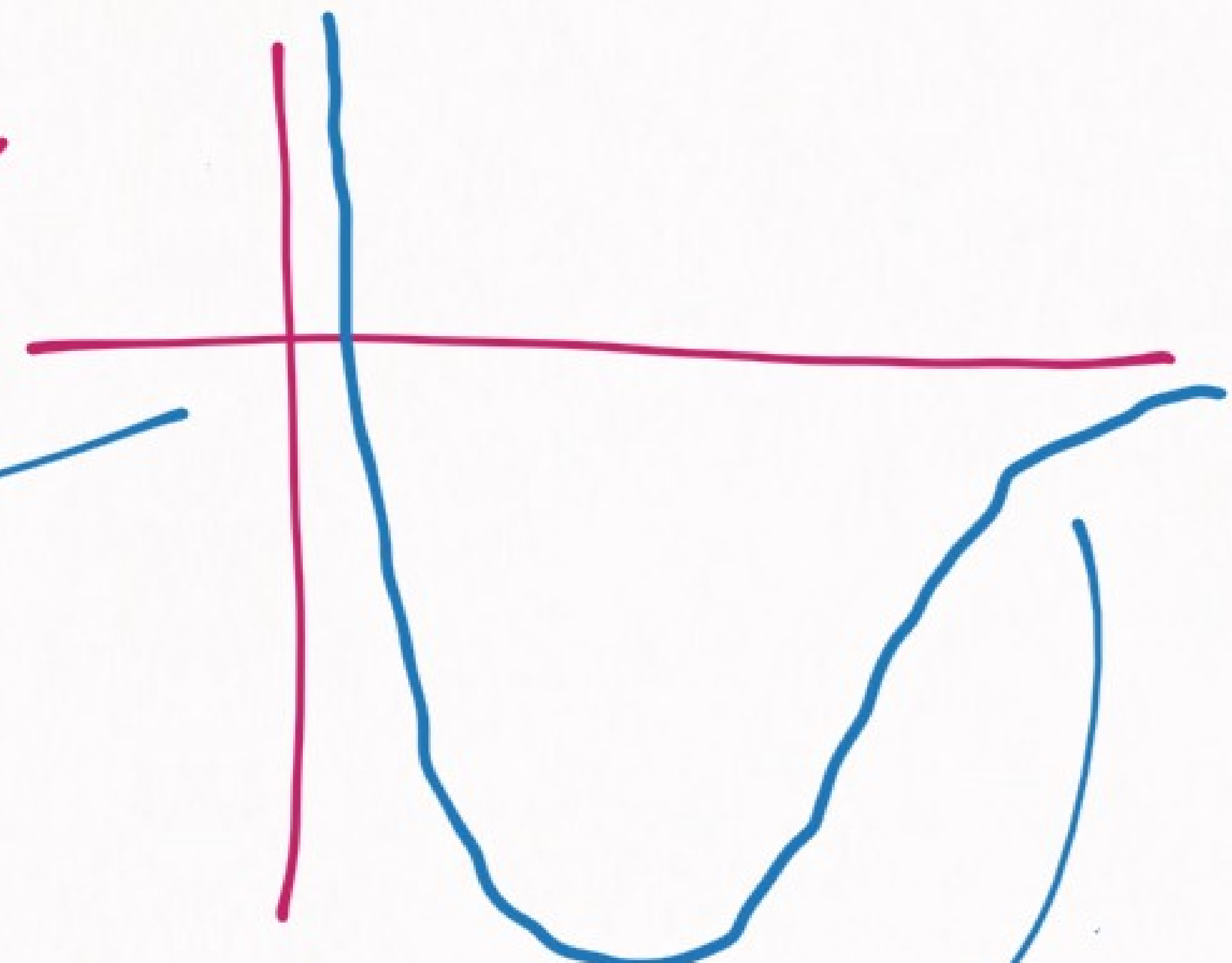
→ we can rely on really braindead estimations of their properties

Example: the boiling point of cesium (铯)

Cs → atomic number 133

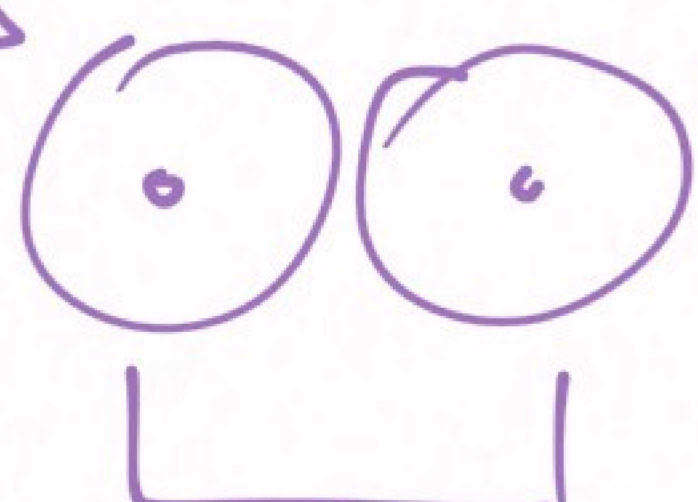
$m_{Cs} \simeq 130\ \text{GeV}$ approximately

Cs - Cs potential is like this:



short-range repulsion

van der Waals tail



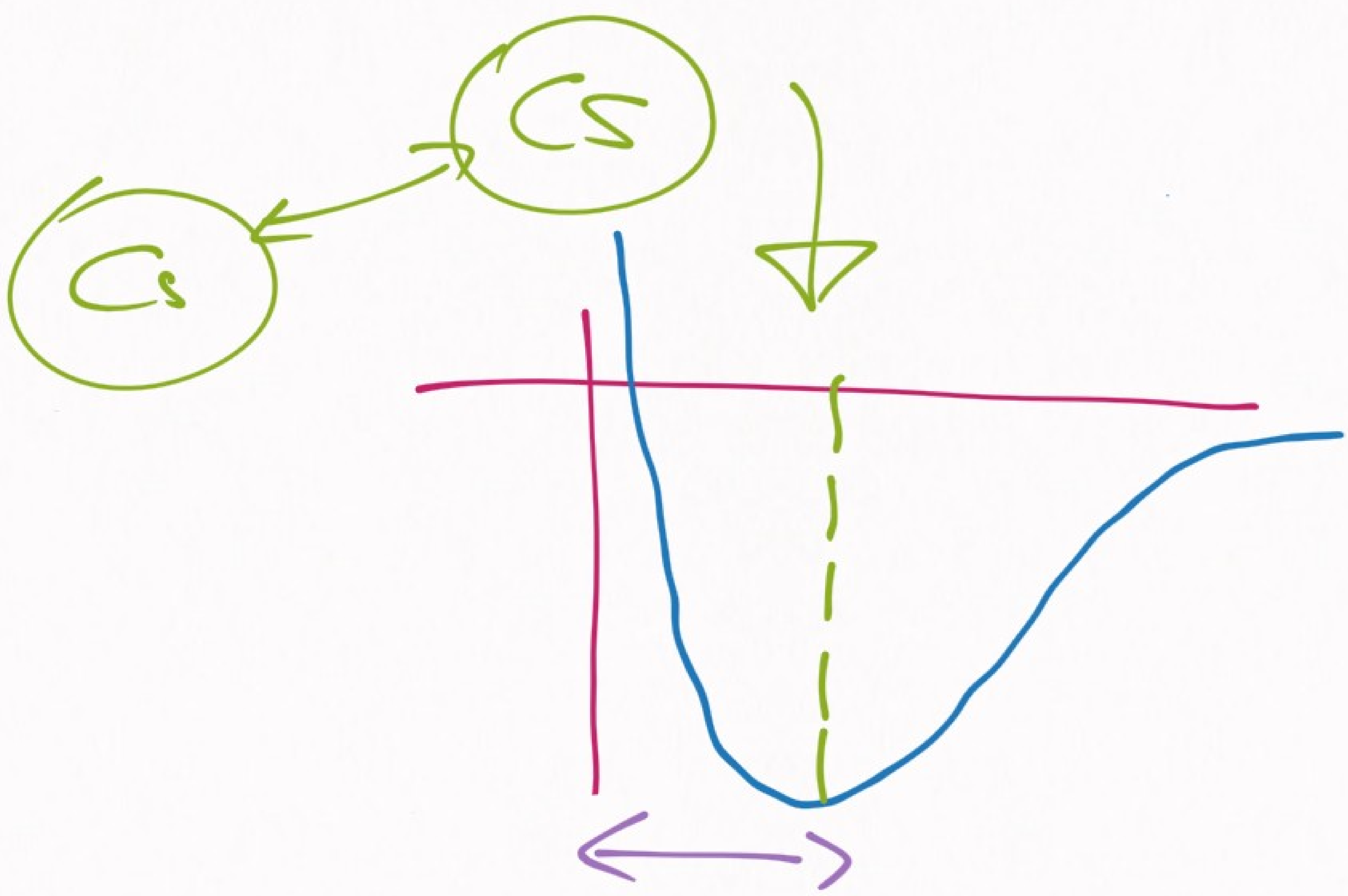
the cesium atoms touch

$\Rightarrow$ repulsion

$$V_{vdW} = -\frac{1}{2m}\frac{R_6^4}{r^6}$$

(van der Waals potential)

Now the Cs atoms will pack around the minimum



standard separation between two Cs atoms

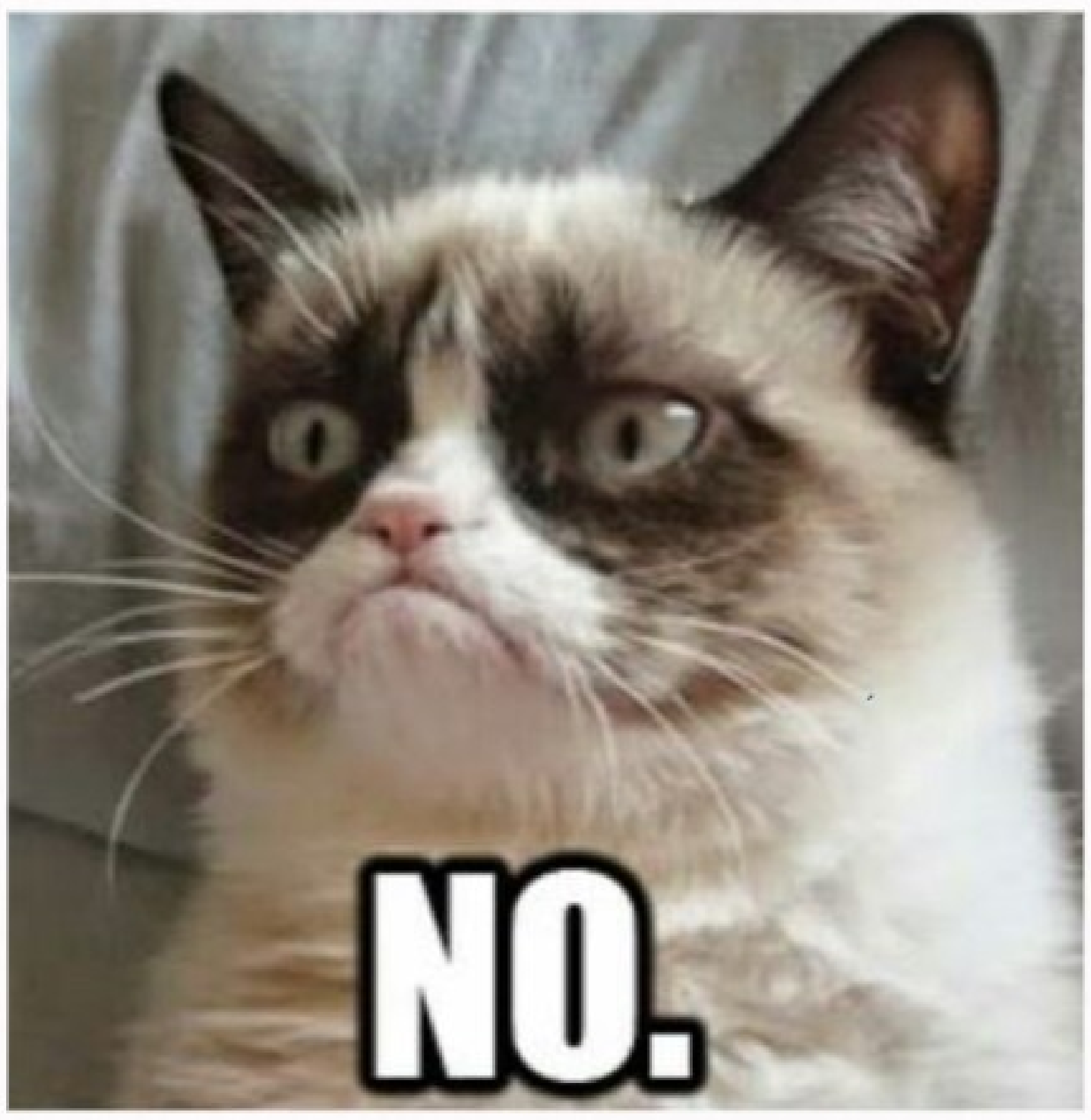
and we can calculate their speed when here

$P \sim Q$ → what is the value of Q ?

$$2mV_{vdW} = -\frac{R_6^4}{r_6} \mapsto \circledast$$

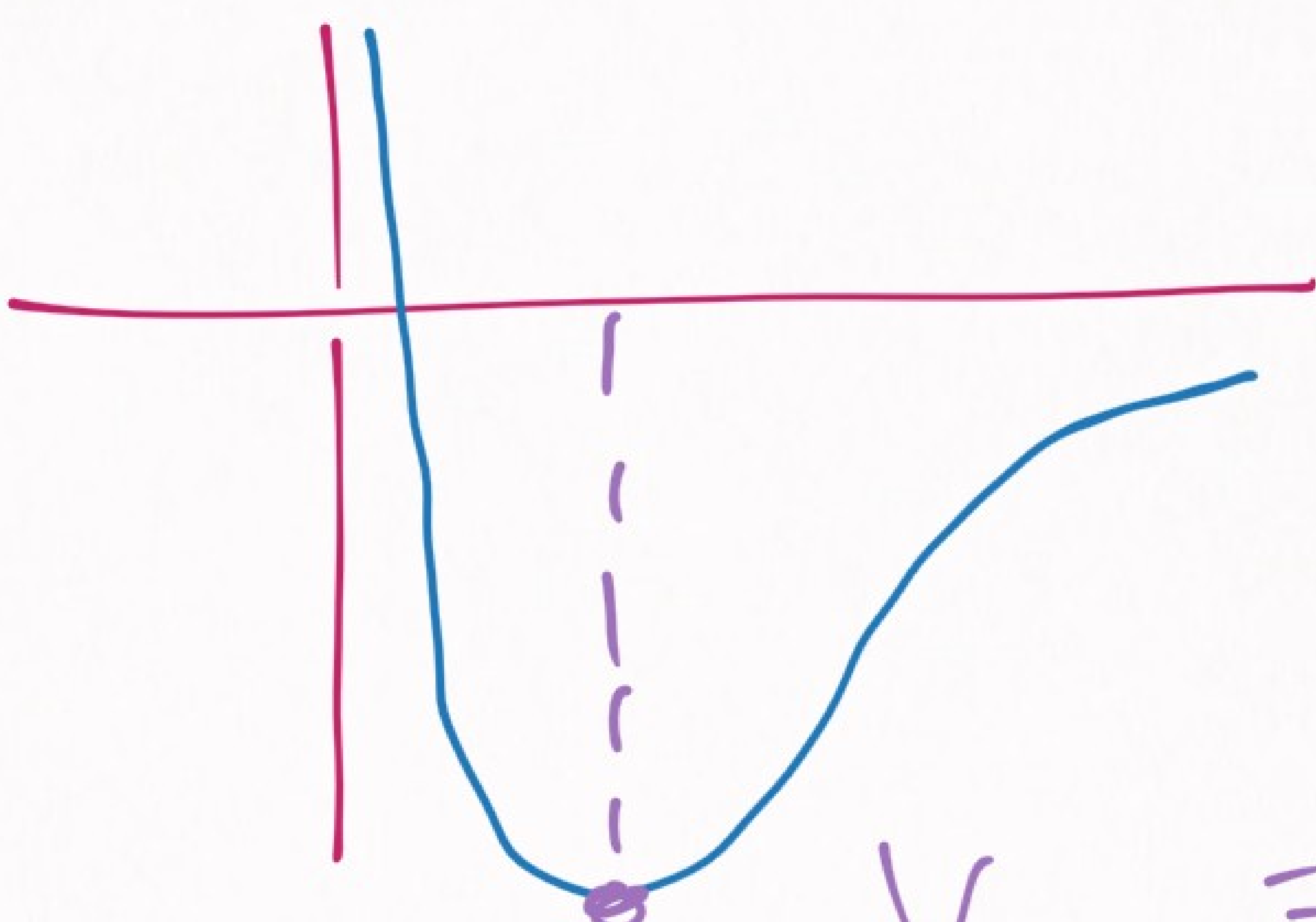
$\circledast$ → maybe $Q \sim \frac{1}{R_6}$?

($R_6 \sim 203\,a_B$, by the way)



Well, you heard Grumpy Cat!

Actually the correct answer has to do with the potential at the minimum

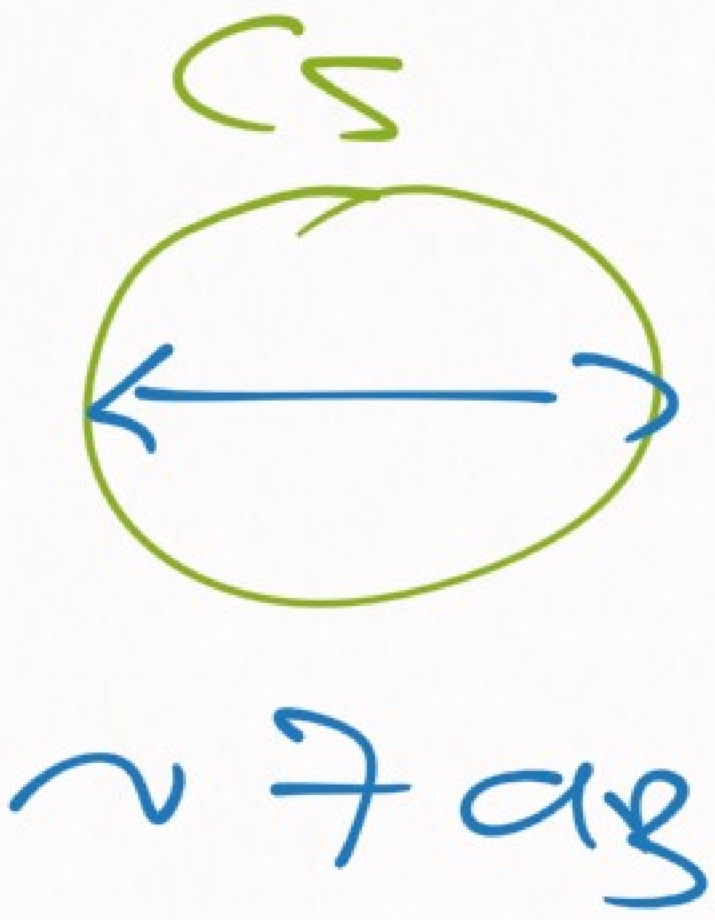


$$V_{min} = -\frac{1}{2m}\left(\frac{1}{R_{Cs}}\right)^2$$

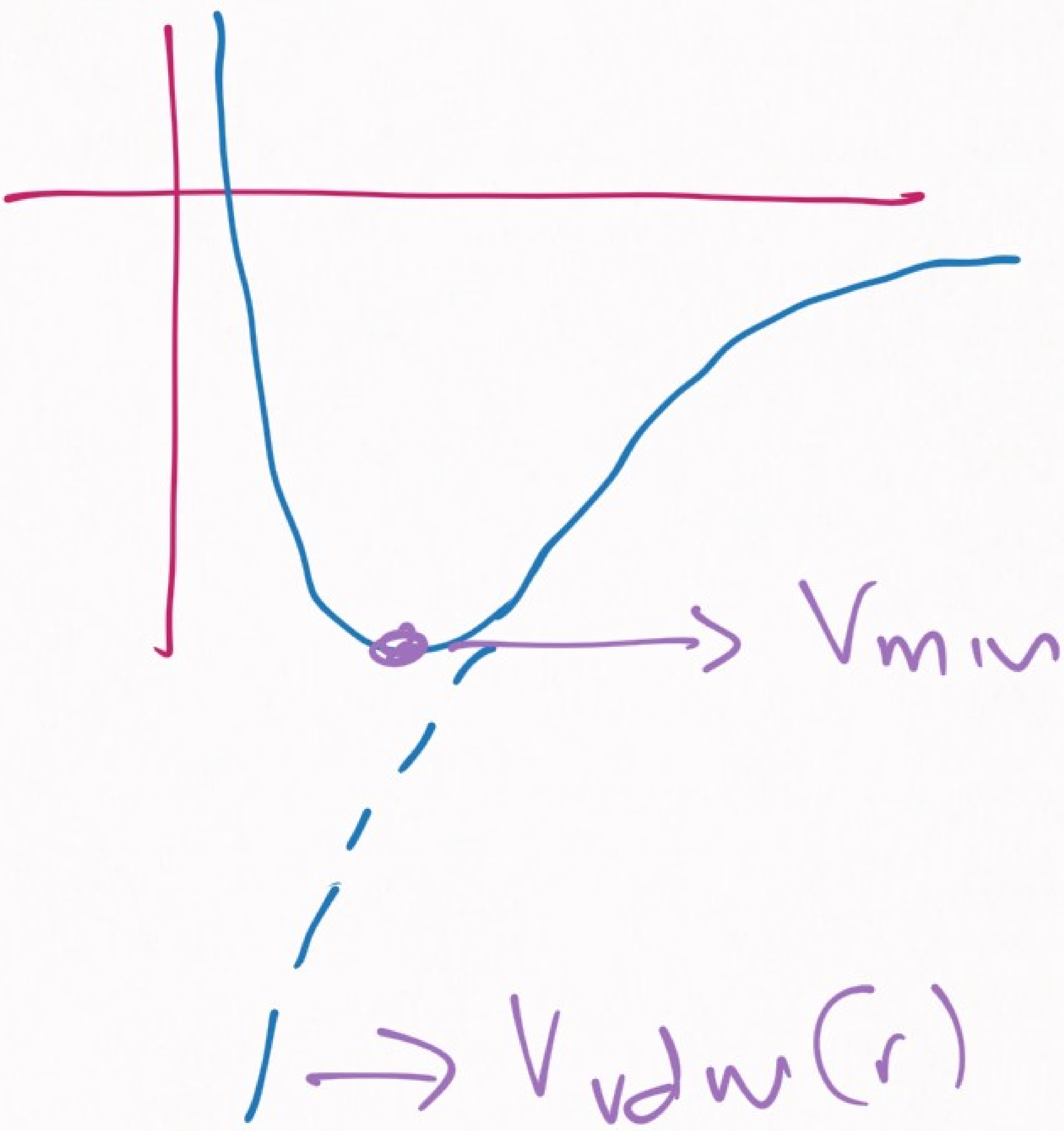
Quick & dirty estimate:

take $V_{vdW}(r = R_s)$,

with R_s a bit bigger than the Cs atom size



so we take $R_s \sim 10 a_B$



$$V_{min} \simeq V_{vdW}(R_s)$$

(差不多!)

$R_S \sim 10\, a_B \qquad R_6 \sim 203\, a_B$

$$V_{vdW} = -\frac{1}{2m}\left(\frac{R_6^4}{r^6}\right)$$

$$V_{min} = -\frac{1}{2m}\frac{1}{R_{cs}^2}$$

$$\Rightarrow R_{cs} \sim R_S\left(\frac{R_S}{R_6}\right)^2$$

$$Q \sim \frac{1}{R_{cs}}$$

$$\Rightarrow v \sim \frac{1}{m} Q \sim 1.16 \cdot 10^{-6}$$

or $\boxed{v \sim 350\, m/s}$

So a group of Cs atoms in liquid form will move at about 350m/s

Boiling point?

→ when the thermal energy of the atoms make them move at about 350 m/s

$$E_{thermal} = \underbrace{K}_{\text{Boltzmann constant}}\overbrace{T}^{\text{temperature}}$$

$$K \simeq \frac{1\ eV}{11000\ \underbrace{K^{\circ}}_{\text{Kelvin}}}$$

We match things and...

$$\frac{1}{2}mv^2 = KT \Rightarrow v = \sqrt{\frac{2KT}{m}}$$

$\Rightarrow$ for $T = 1000\,K^\circ$

$$v \simeq 350\ m/s$$

Expectation: liquid Cs should boil at about $1000\,K^\circ$

Reality: $944\,K^\circ$

$$\boxed{\text{NOT BAD!}}$$

To summarize:

[Naturalness allows us to chabuduo physics easily]

And this is the reason why atomic physics is easy but nuclear physics is difficult

(because nuclear physics is not natural)

BUT... (there's always a bit)

$$\boxed{\text{most problems involve several scales}}$$

→ this will entail corrections

Hydrogen atom:

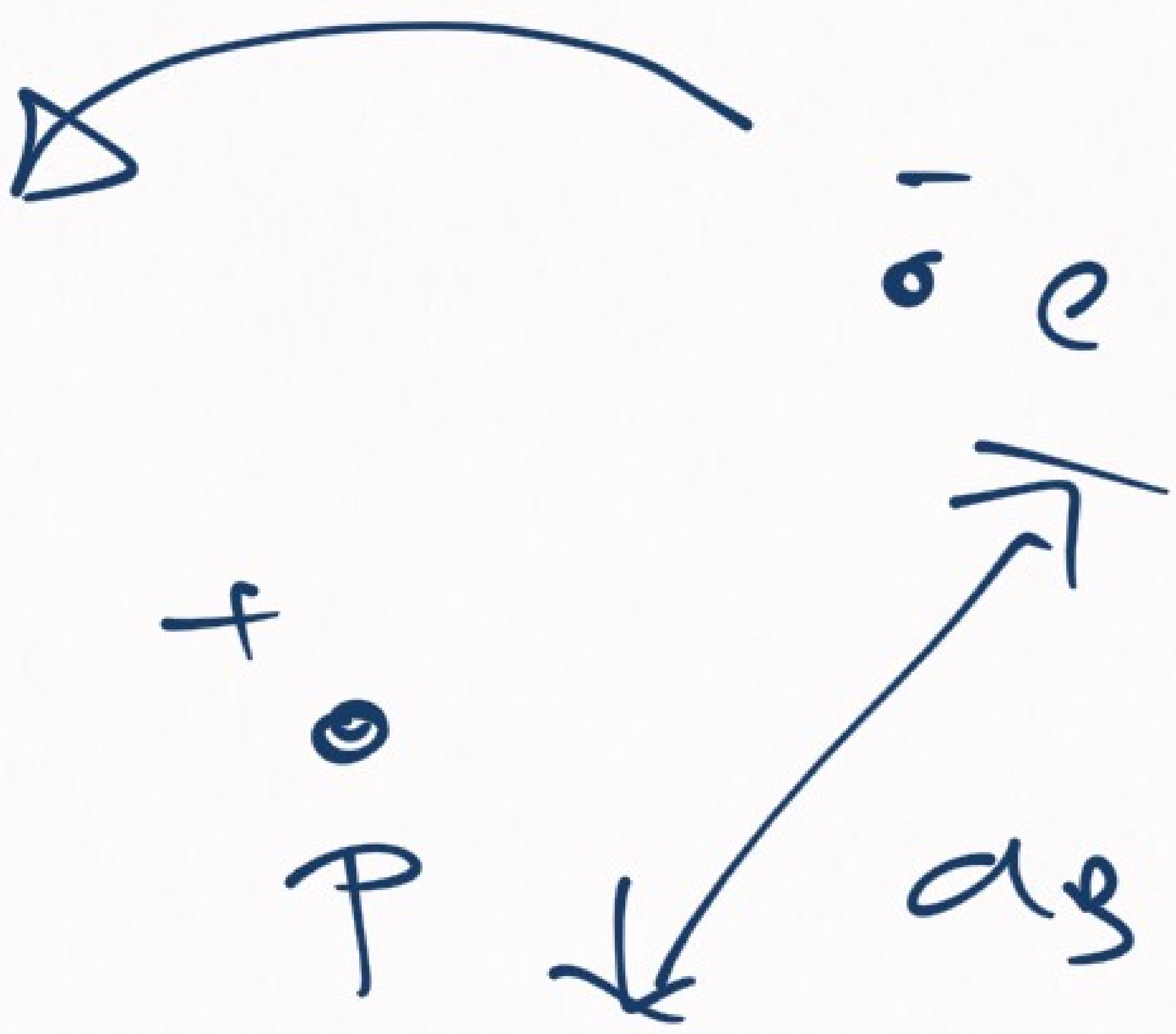
1) most importan scale

$$a_B = \frac{1}{m_e \alpha}, \quad Q_B = m_e \alpha$$

2) other scales:

$r_e = \frac{1}{m_e}, \; r_p$ ⟶ electron's proton size

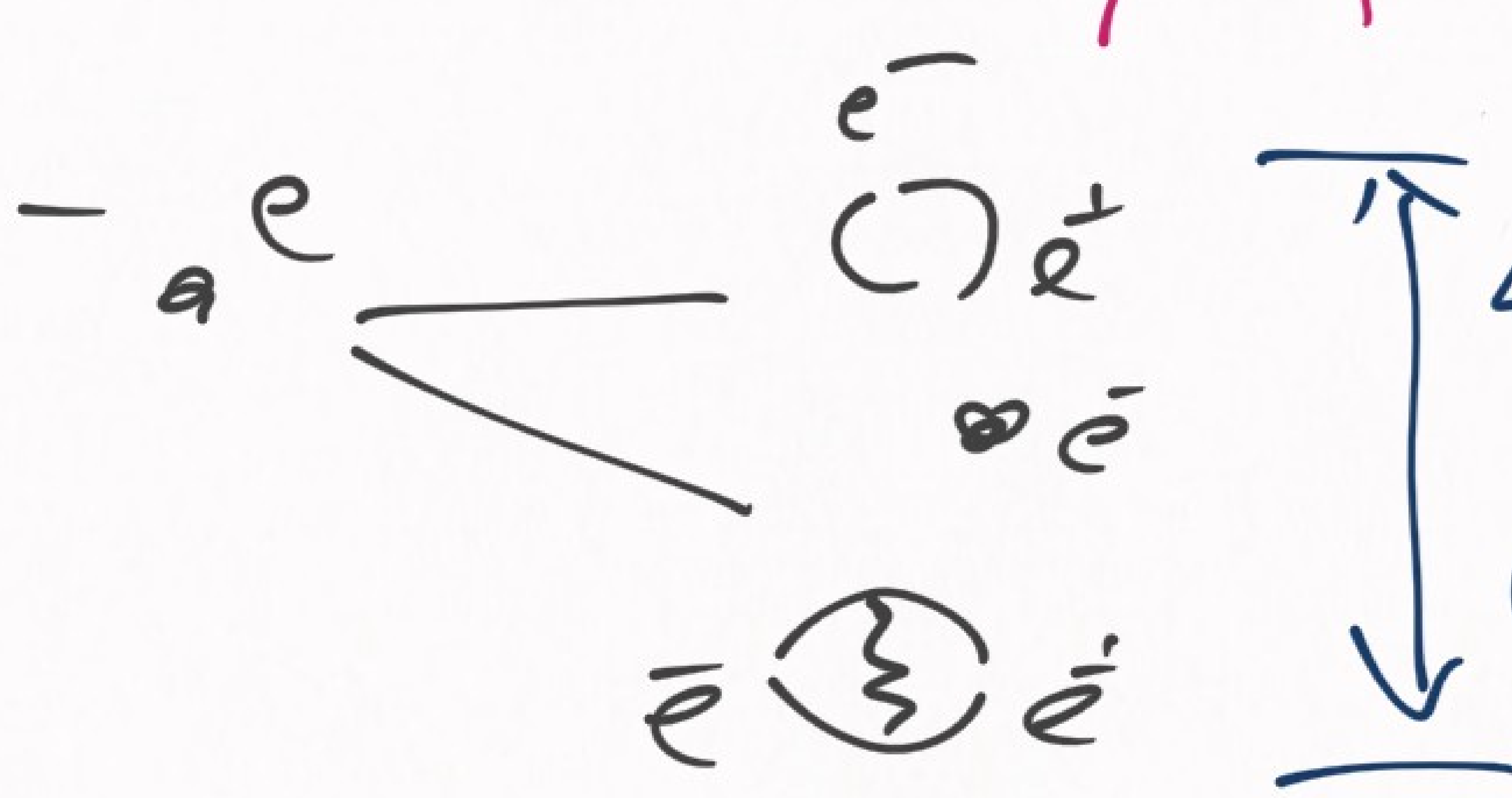
On the one side we have:



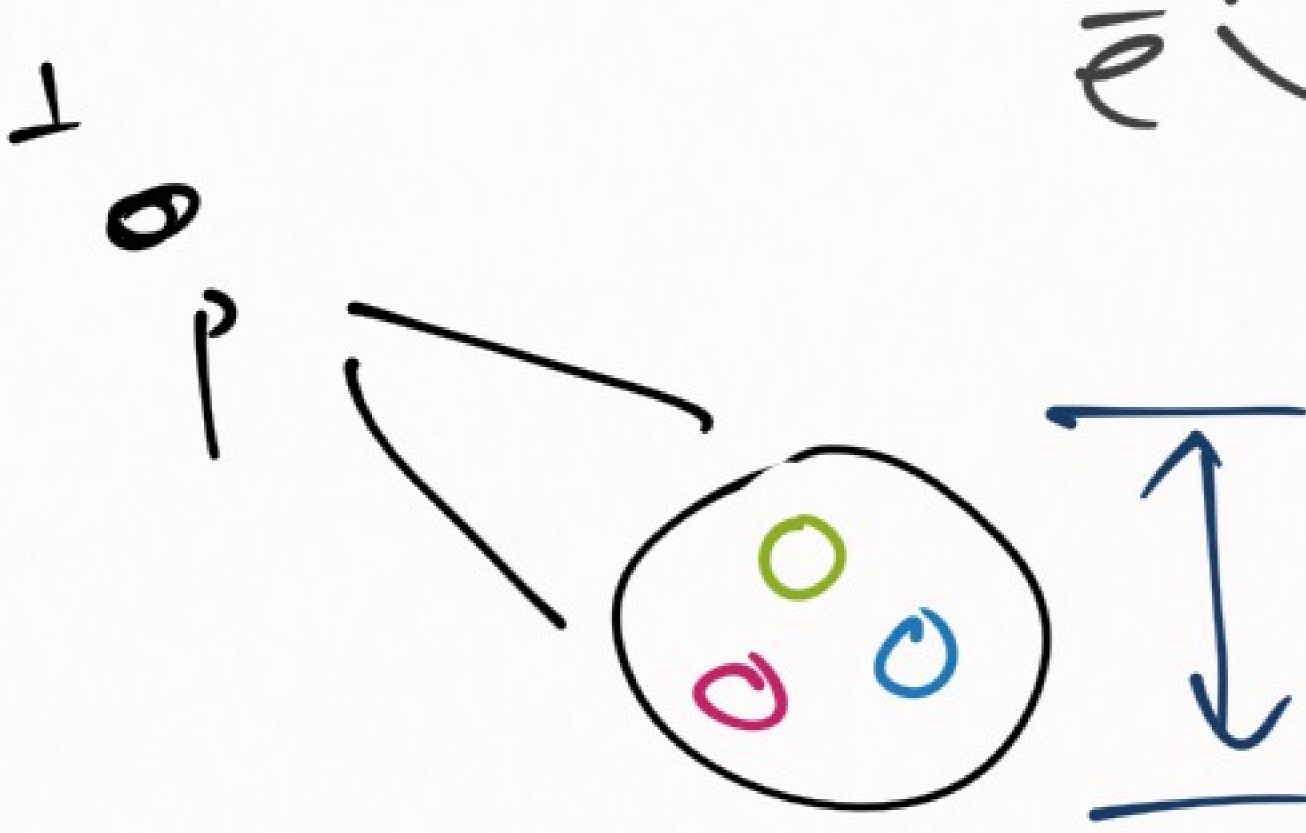
$a_B \simeq 5.4 \cdot 10^4\ \mathrm{fm}$

But zooming in:

vacuum polarization



400 fm $\left(\frac{1}{m_e}\right)$



1 fm

↳ internal structure of proton

That is, we have several scales:

$a_B \sim 54000$ fm

$\frac{1}{m_e} \sim 400$ fm

$r_p \sim 1$ fm

But there's a separation of scales:

1) $$\frac{r_e}{a_B} = \frac{m_e \alpha}{m_e} = \alpha \simeq \frac{1}{137}$$

2) $$\frac{r_p}{a_B} \simeq \frac{1}{50000} \simeq 2 \cdot 10^{-6}$$

Thus, we can expect small corrections from these smaller scales

1) electron structure

→ corrections of relative size $\frac{r_e}{a_B} \sim \frac{1}{137}$

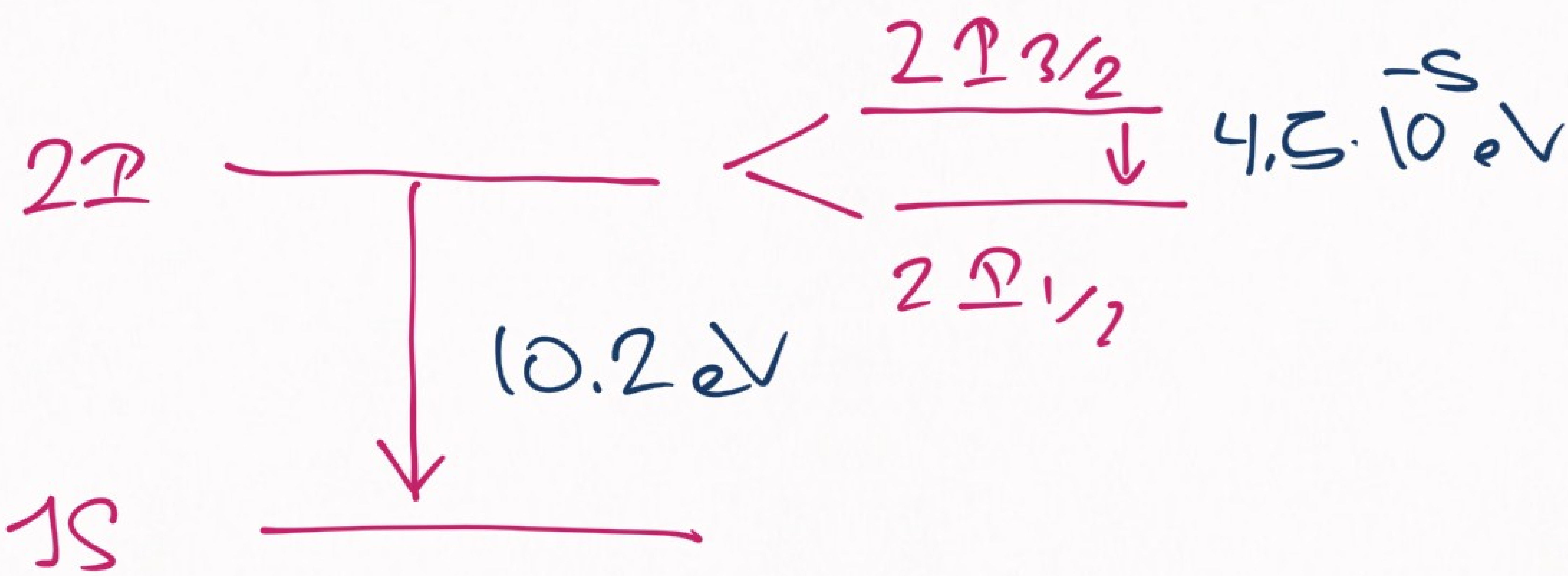
→ but this is a relativistic effect, it would be more correct

$$\left(\frac{r_e}{a_B}\right)^2 \sim \frac{1}{19000} \sim 5 \cdot 10^{-5}$$

Expectation for the correction

$$\boxed{5\cdot 10^{-5}}$$

Reality: fine structure corrections



$$\frac{4.5\cdot 10^{-5}\,\text{eV}}{10.2\,\text{eV}} \sim \boxed{5\cdot 10^{-6}}$$

Not perfect, but not that bad either

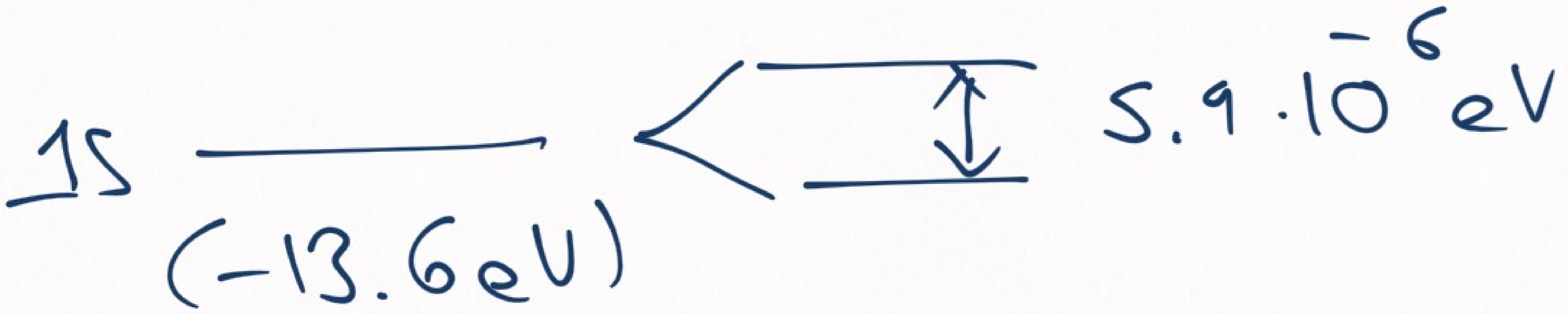
2) Proton size corrections:

$$\boxed{\left(\frac{r_p}{a_B}\right) \sim 2 \cdot 10^{-6}}$$

Expectation

(different from electron size correction: proton size comes from internal structure)

Reality:



$$\frac{5.9 \cdot 10^{-6}\,\text{eV}}{13.6\,\text{eV}} \sim \boxed{5 \cdot 10^{-7}}$$

Not bad

Lesson → Systems that

1) are natural
2) have a good separation of scales

are a piece of cake

Example: hydrogen atom

→ We can do incredibly accurate predictions even in undergraduate courses

Corolary: systems that

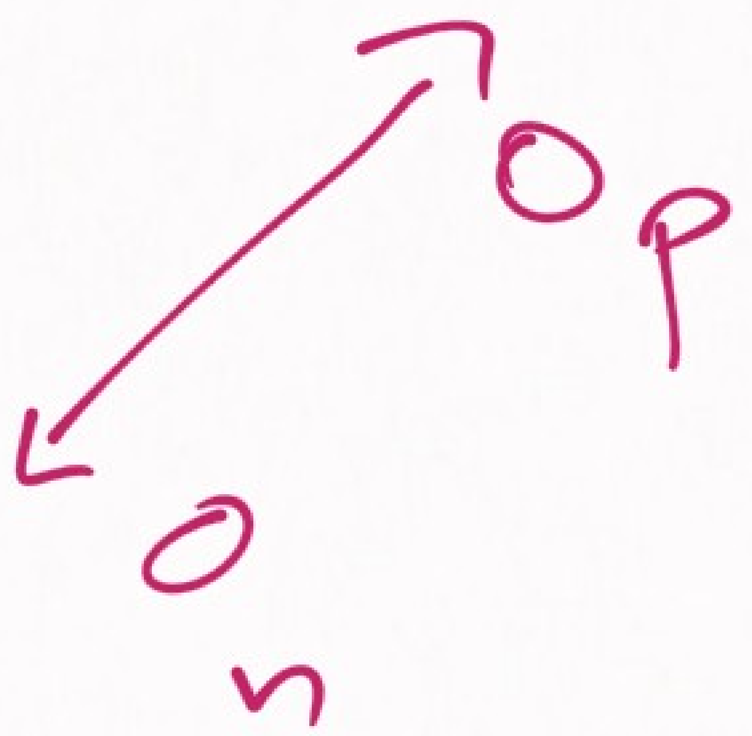
1) are not natural
2) have a poor separation
   of scales

are a nightmare

However they are wonderful as research projects

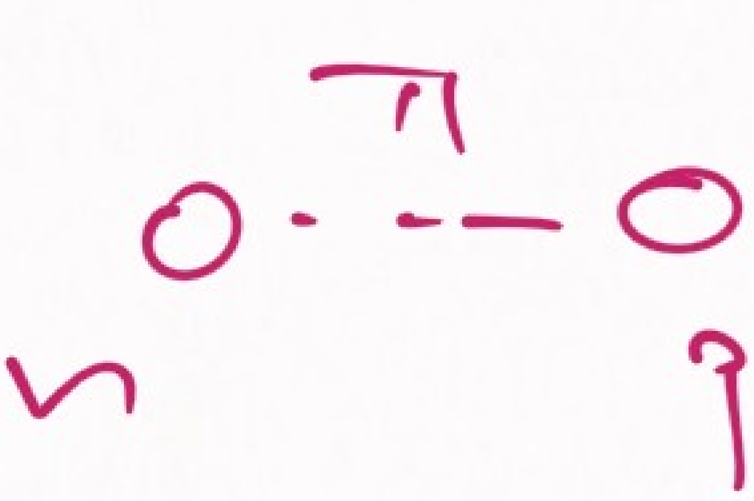
What about nuclear physics?

1) Deuteron size



$r_d \sim 5$ fm

2) Range of nuclear force



$\frac{1}{m_\pi} \sim 1.4$ fm

3) Size of nucleons ~ 1 fm

1+2) not too natural
(but not that bad)

2+3) poor scale separation

And this is why nuclear physics is difficult

Welcome to the wonderful world of nuclear physics!