NUCLEAR PHYSICS ①

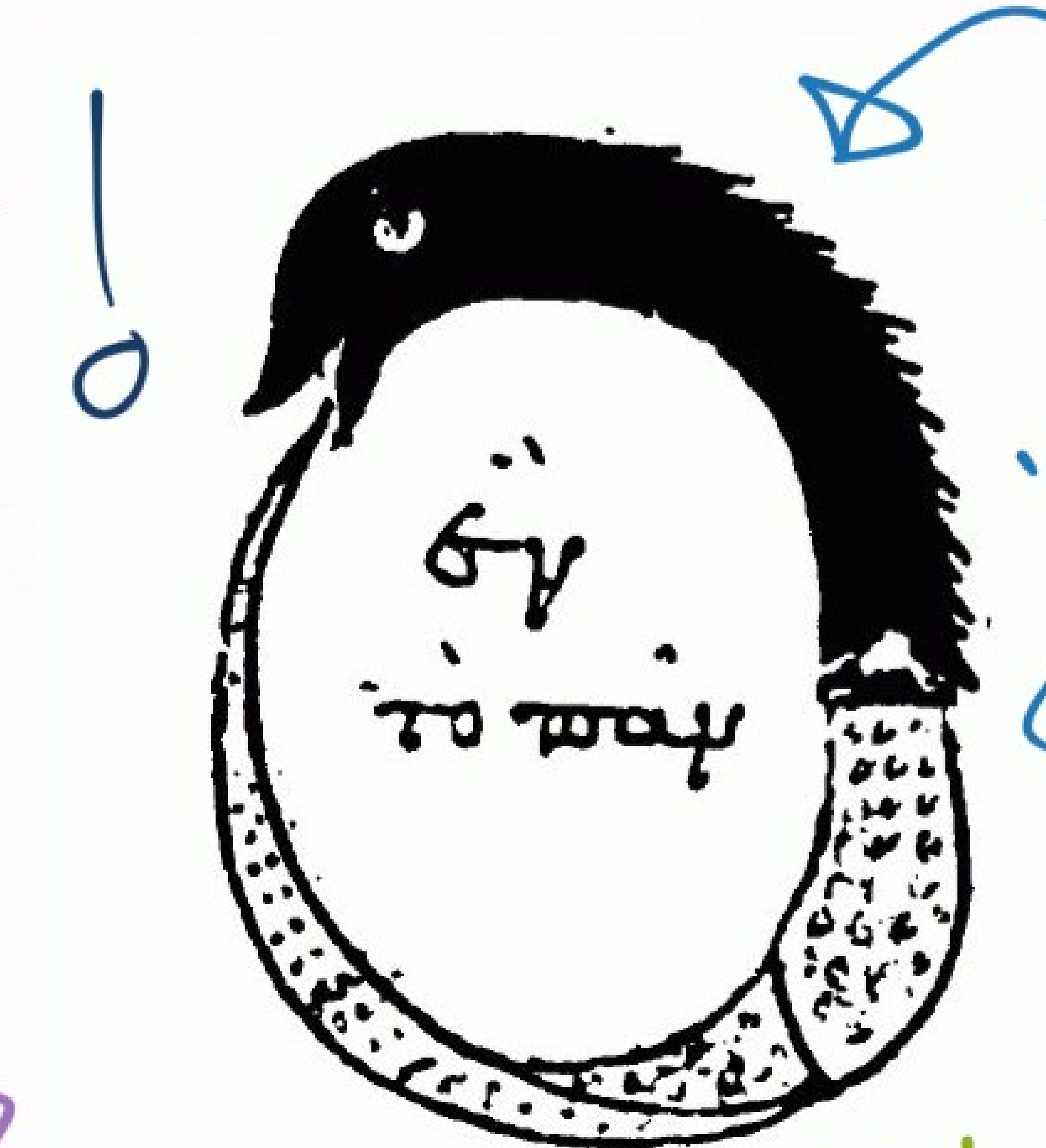
"A matter of scales"
(尺度的问题)

Our understanding of nature depends on the scale (尺度)

from the very big to the very small

Each scale → different understanding

Let's do a tour!



← ouroboros

"εν το παν"

(all is one)

42!

∞

↓

metaphysics (形而上学) =D

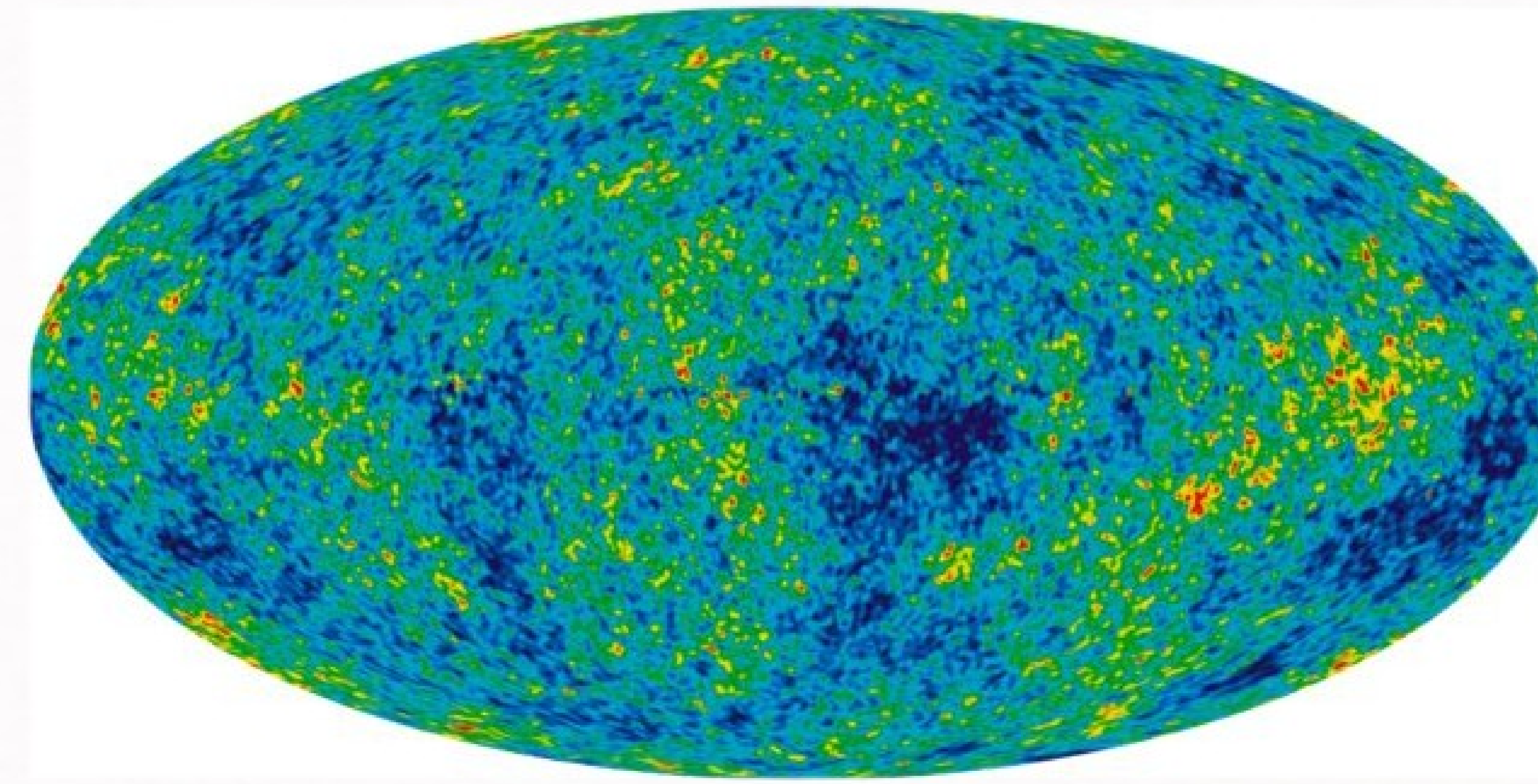
道可道

非常道

The multiverse?

⟹ 10⁹ light years

⟹



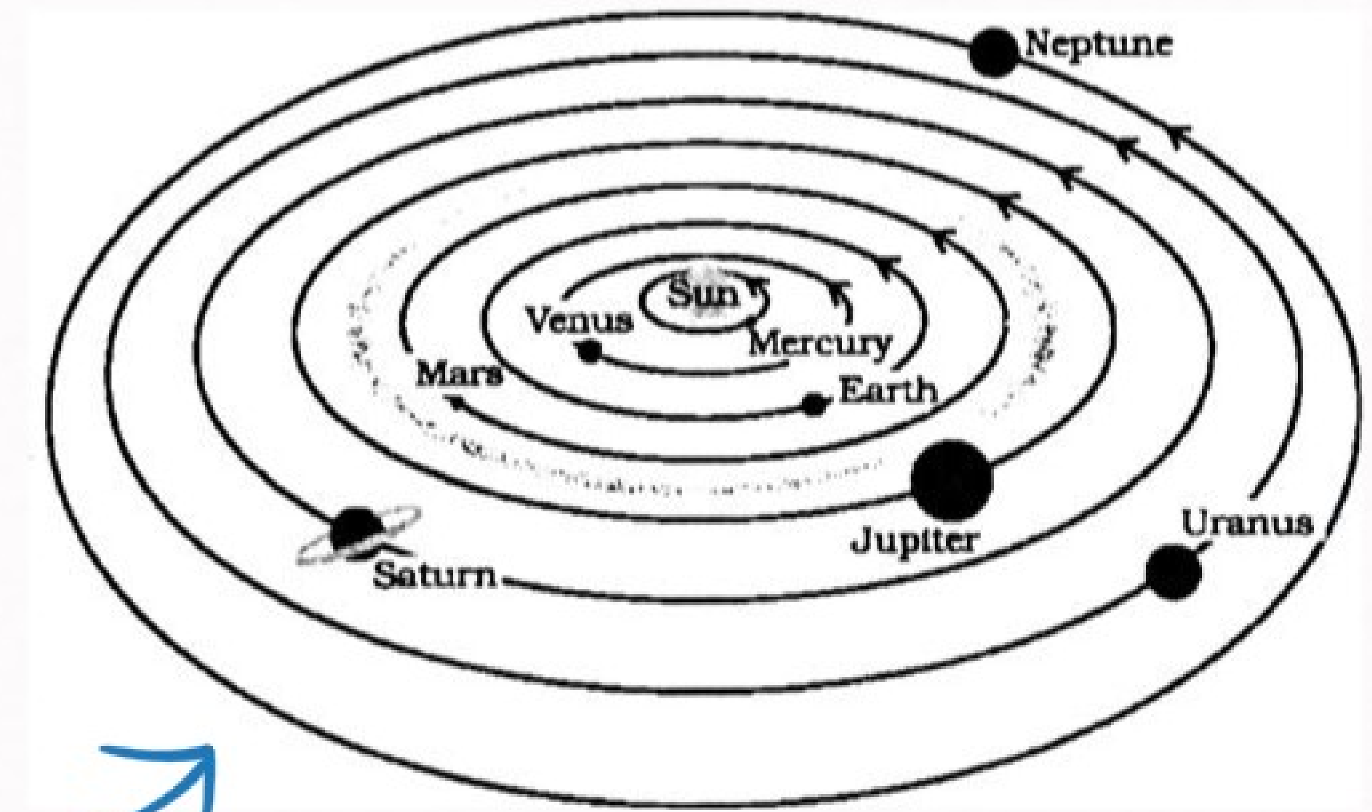
cosmology

not-so-many light years

⟹

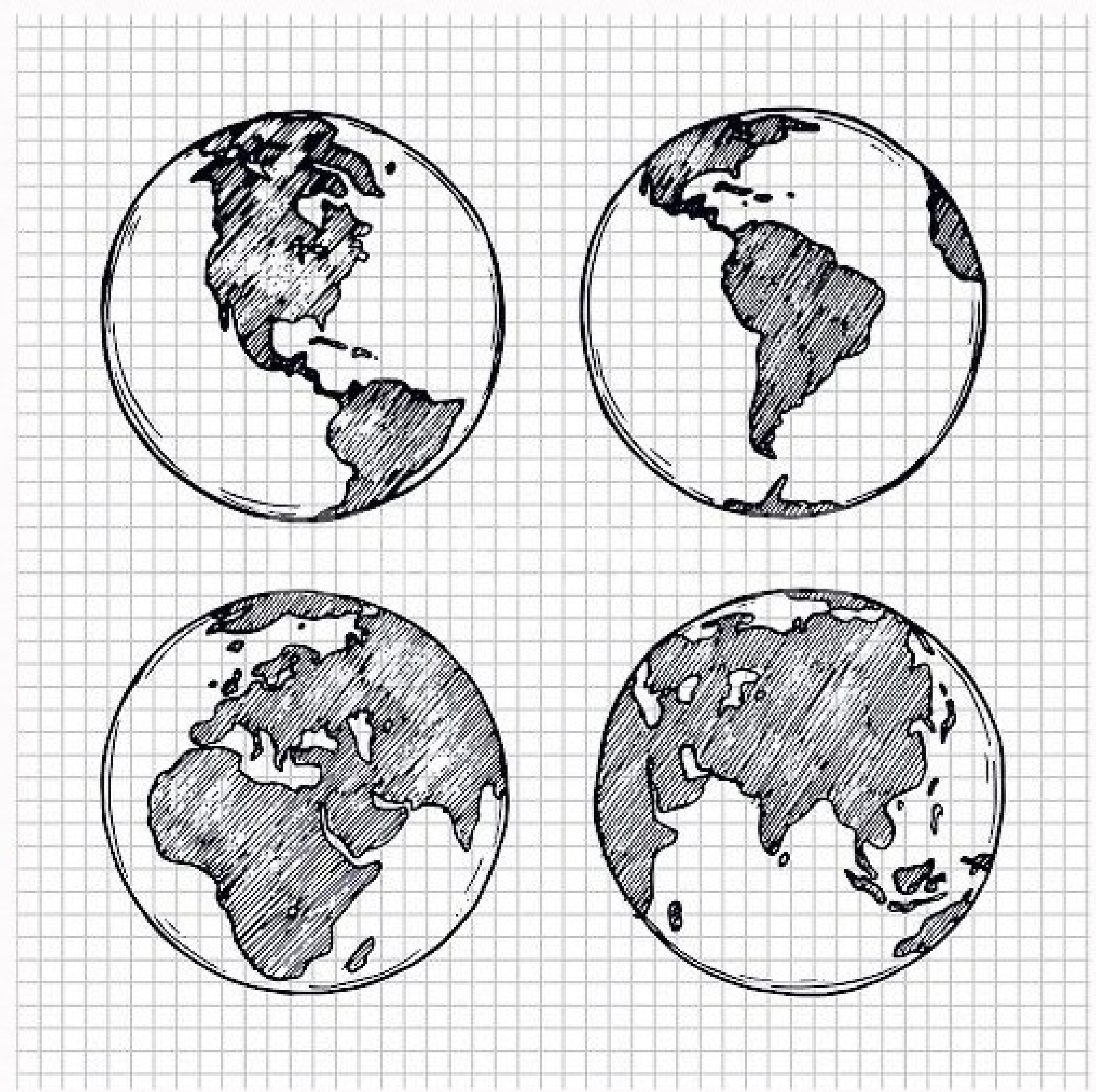
astrophysics

100 astronomical units



(mostly) classical mechanics

10^4 km



geography
ecology
geopolitics

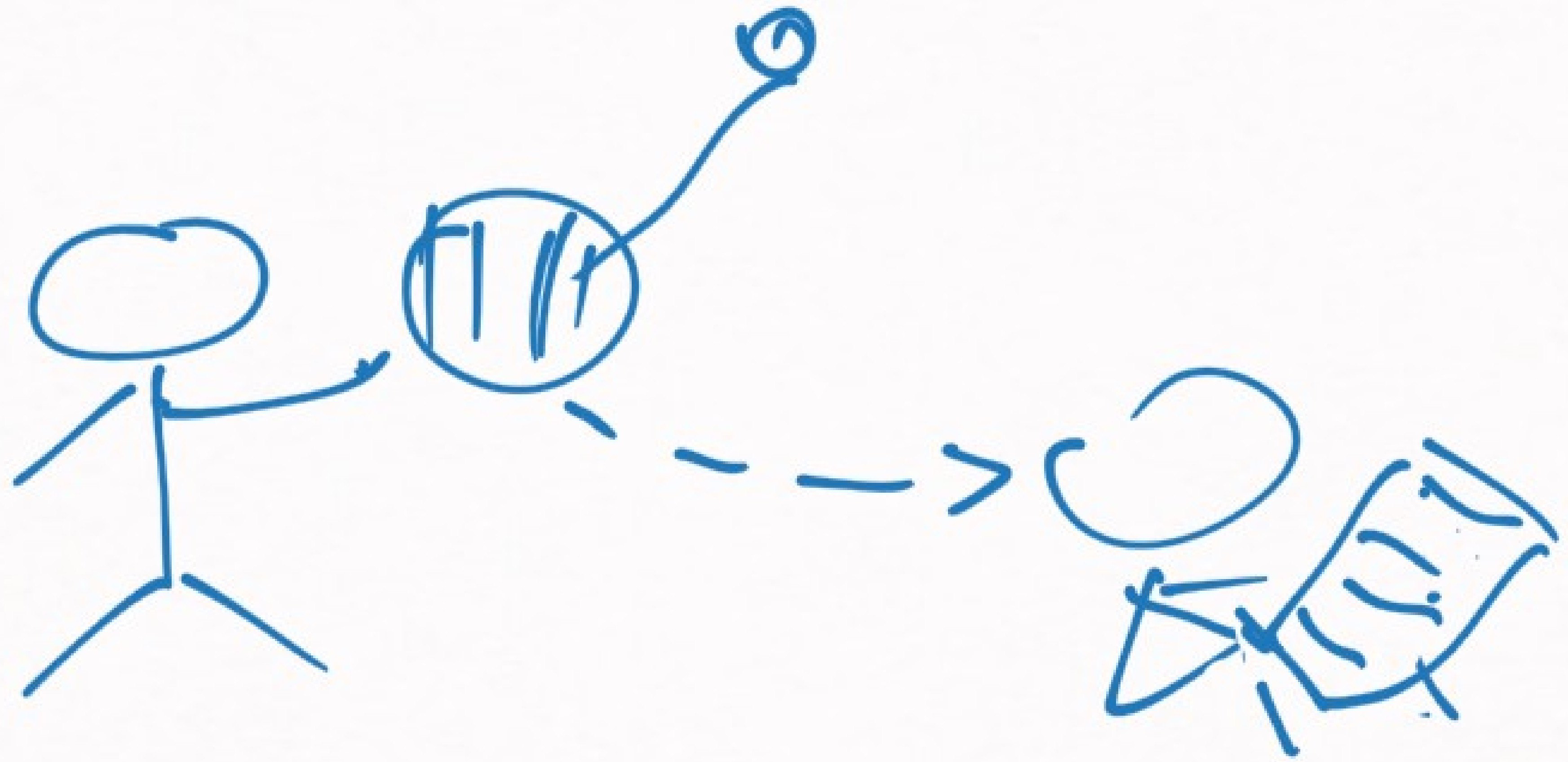
10^3 km

politics
sociology
economics

1 km

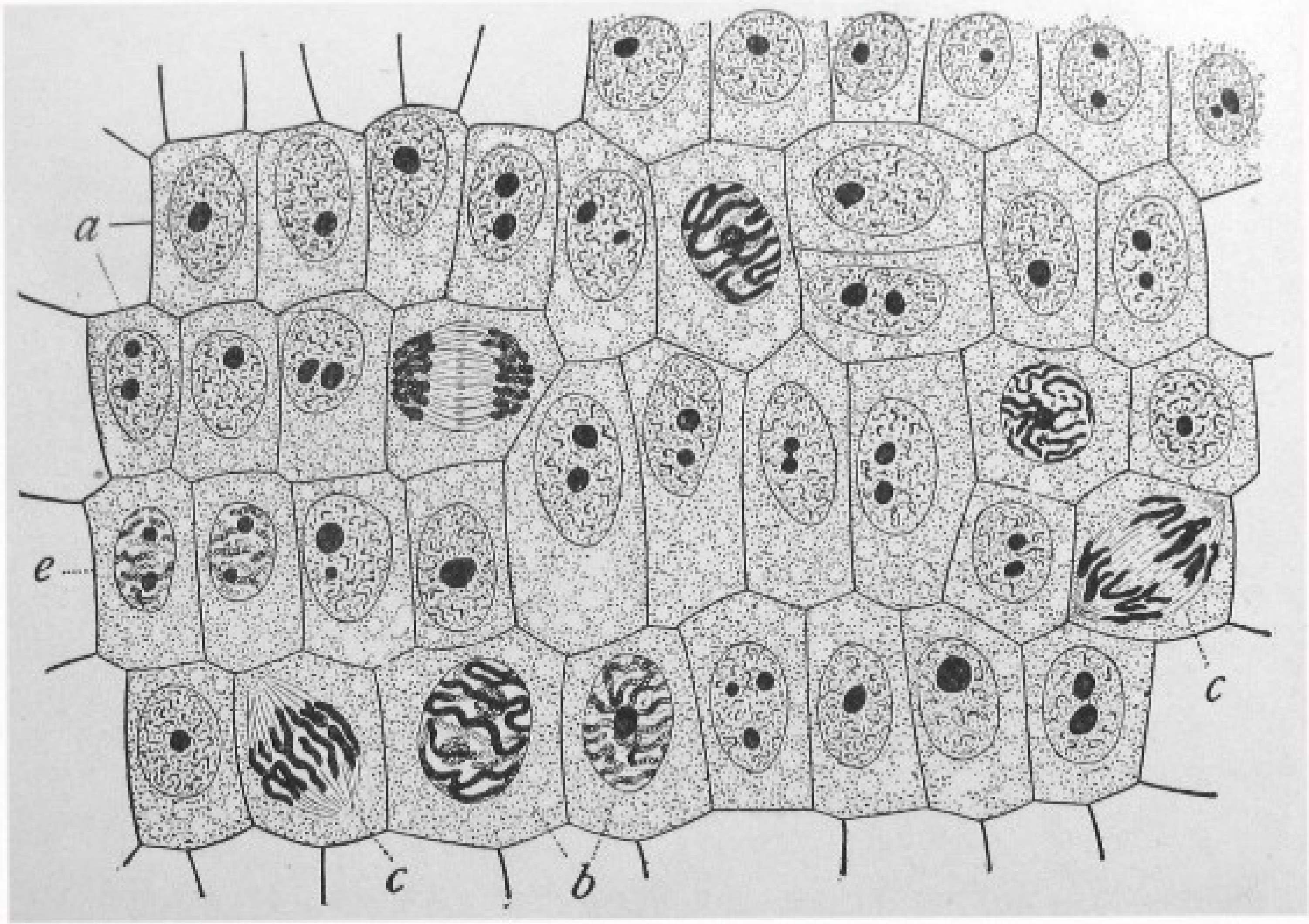
social life

1 m



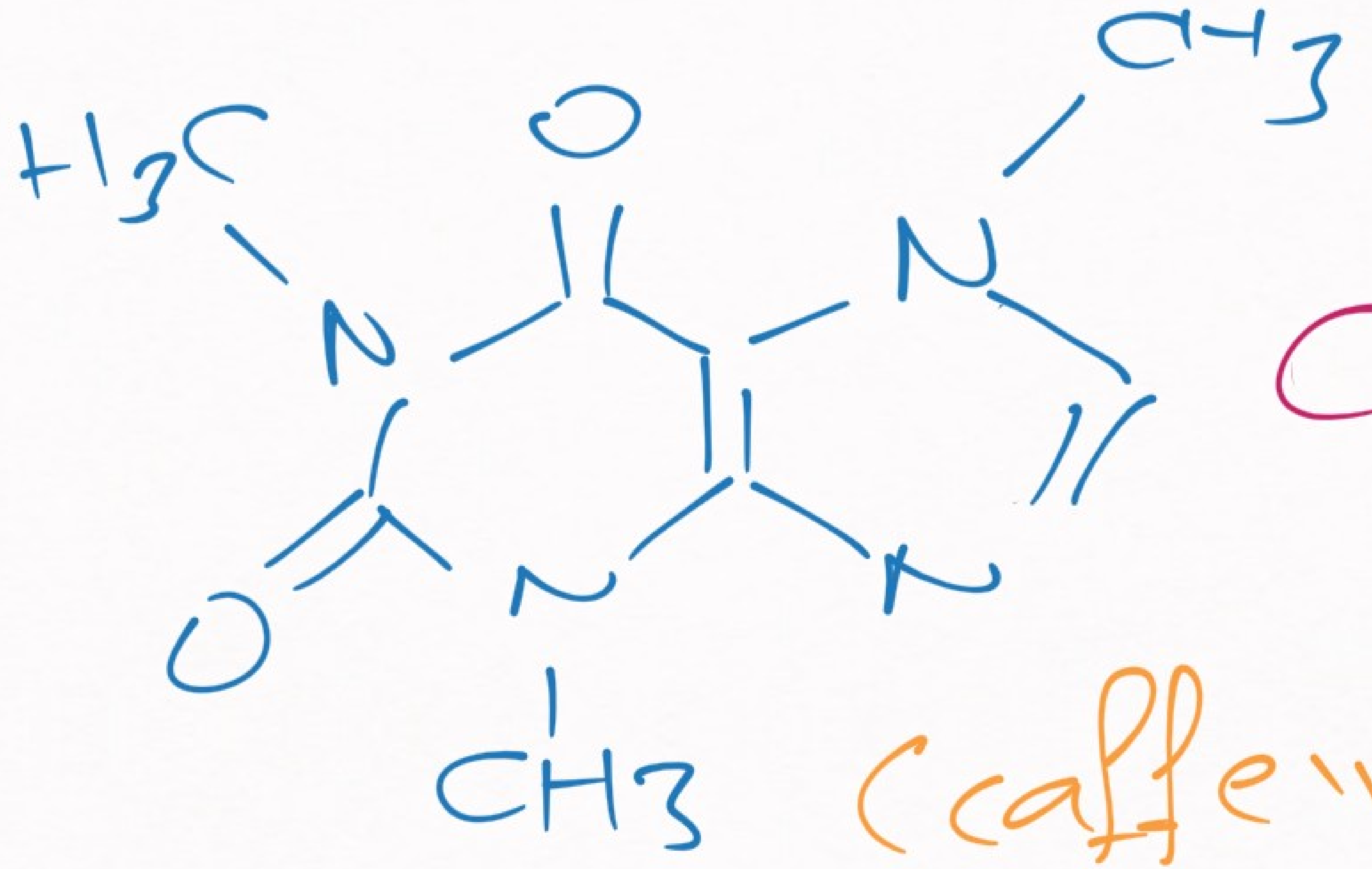
classical mechanics (I guess)

10^{-5} m



biology

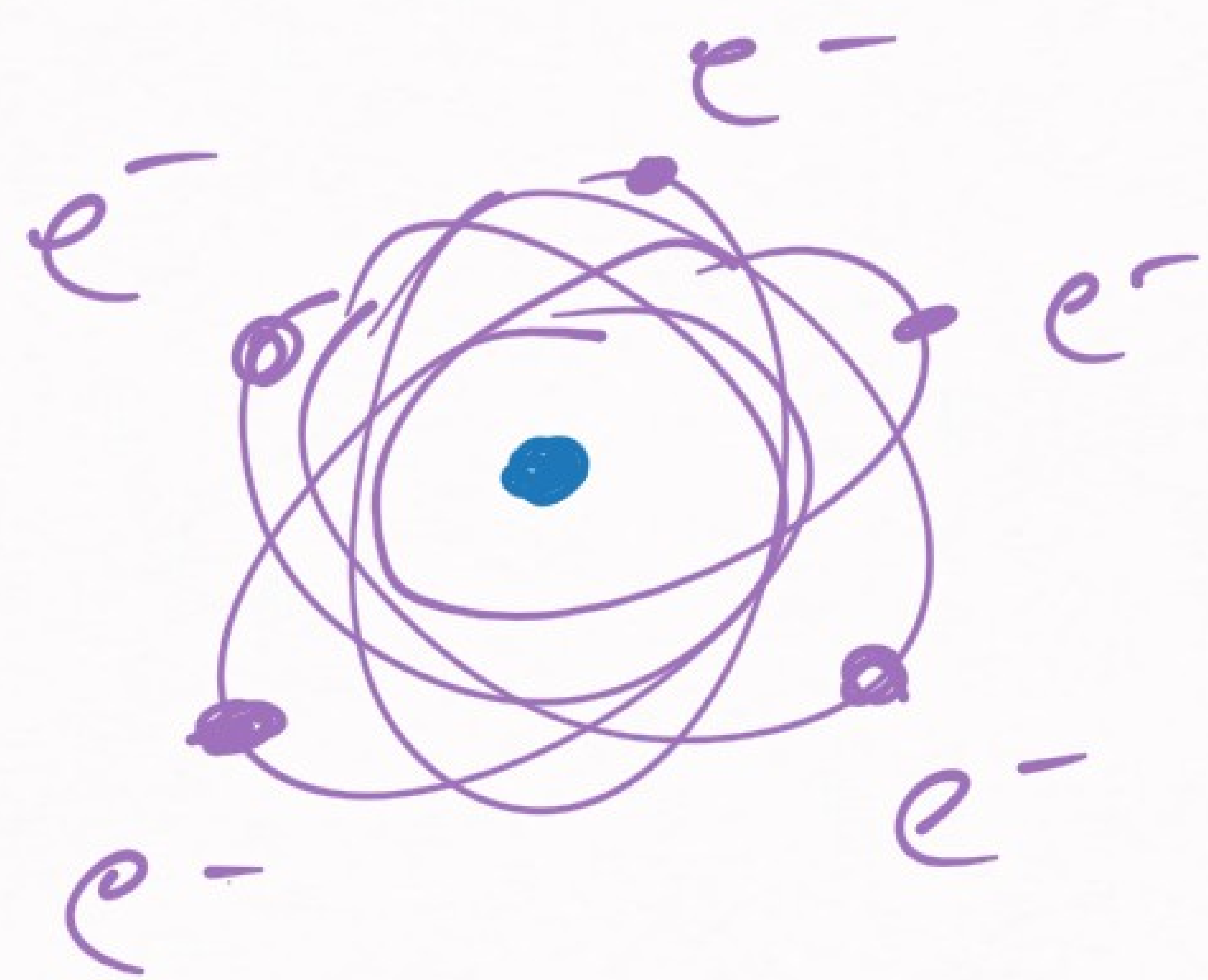
10^{-9} m



chemistry

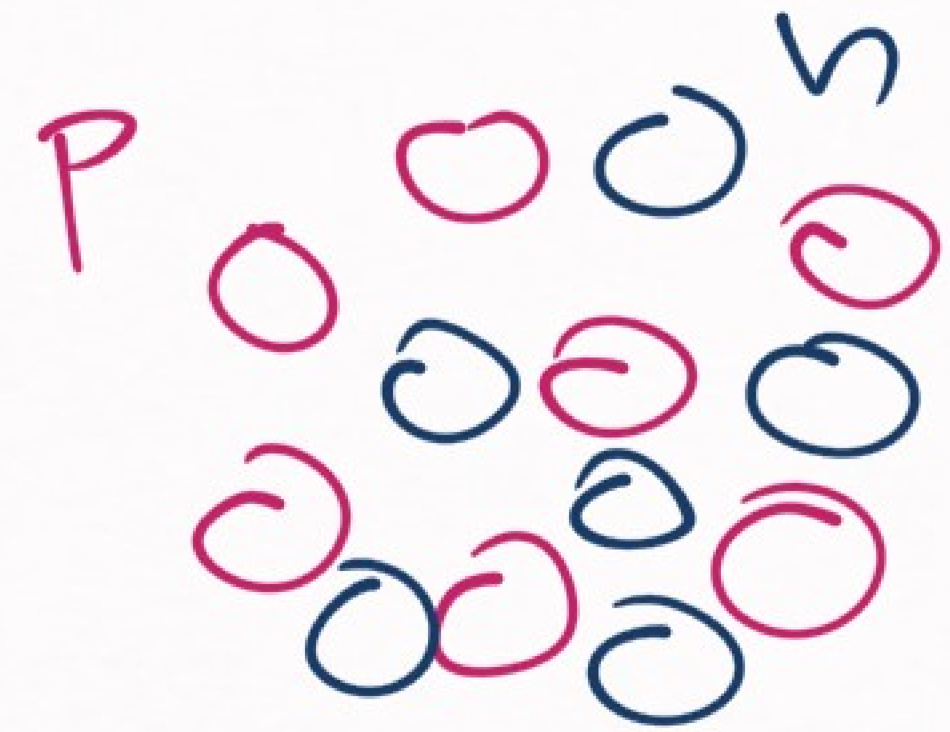
(caffeine)

10^{-10} m



atomic physics

10^{-15} m



nuclear physics

proton   neutron

10^{-16} m

quarks & gluons

quantum chromodynamics (QCD)

$\Rightarrow$ 10^{-35} m Planck scale & quantum gravity

$\Downarrow$

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42!

εν το παν

→ back to metaphysics

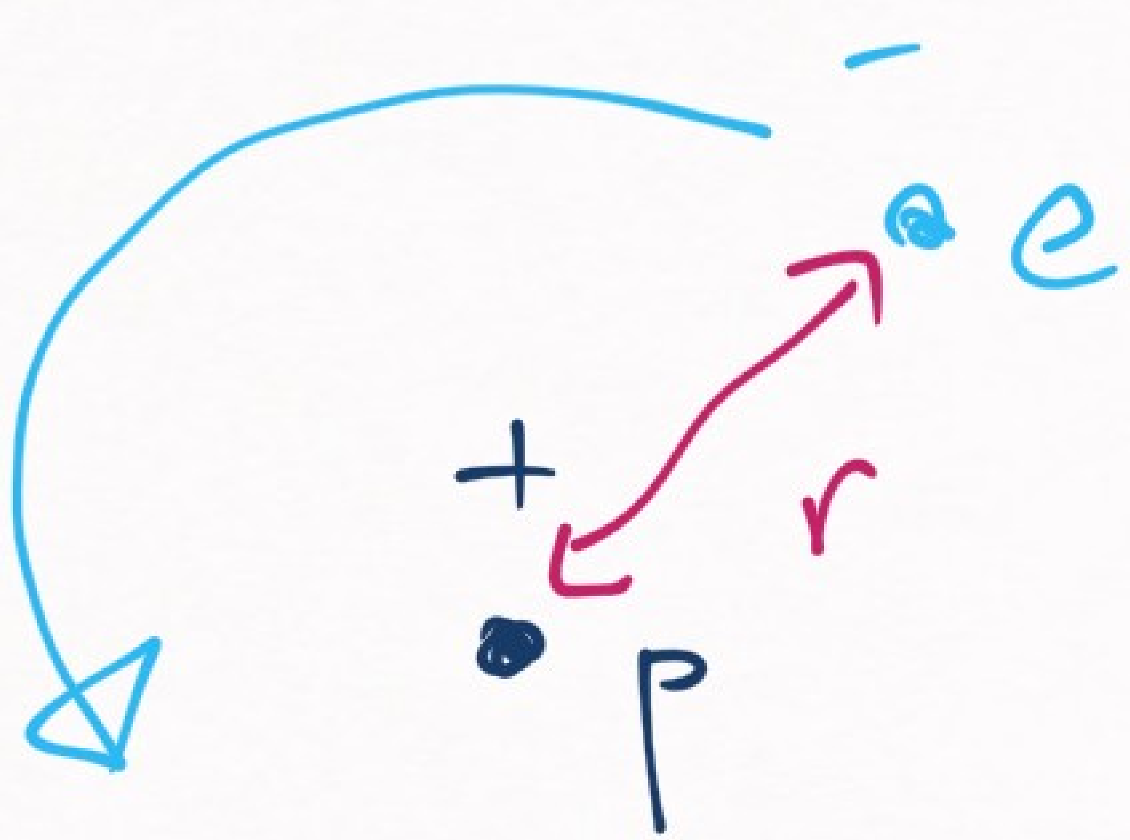
道可道,非常道

→ Here we will worry about the $10^{-(15-16)}$ m scales

(nuclear physics & QCD)

BUT FIRST → Let's go back to atomic physics

THE HYDROGEN ATOM
(FROM THE SCALE VIEWPOINT)



→ They interact via the Coulomb force

$$V(r) = -\frac{\alpha}{r}$$

$\alpha \sim \frac{1}{137}$

r → radius

Let's write the Schrödinger equation:

$$\left[-\frac{1}{2\mu}\vec{\nabla}^2 + V(\vec{r})\right]\psi(\vec{r}) = E_B\,\psi(\vec{r}) \quad \circledast$$

2μ → reduced mass ⇒ $\frac{1}{\mu} = \frac{1}{m_e} + \frac{1}{m_p}$

m_e: electron mass, m_p: proton mass

$\circledast \rightarrow E_B = -\frac{\gamma_B^2}{2\mu}$

γ_B → wave number

Now we modify this a bit:

$$\left[-\nabla^2 - \frac{2\mu\alpha}{r}\right]\psi(\vec{r}) = -\gamma_B^2\,\psi(\vec{r})$$

INPUT: $\frac{2\mu\alpha}{r}$

OUTPUT: γ_B^2 → we want to predict this!

Trick: $\mu\alpha = \frac{1}{a_B}$

a_B: Bohr scale

Explanation:

$$[\text{energy scale}]^{-1} = [\text{length scale}]$$

$\mu\alpha$ $\quad$ a_B

$$\frac{1}{\mu} = \frac{1}{m_e} + \frac{1}{m_p} \simeq \frac{1}{m_e}$$

$$a_B = \frac{1}{\mu\alpha} \simeq 268\ \text{MeV}^{-1}$$

$m_p \gg m_e$ $\quad$ $\begin{cases} m_e \simeq 0.5\ \text{MeV} \\ m_p \simeq 940\ \text{MeV} \end{cases}$

A note about units $\rightarrow$ we are using natural units

$\hbar = 1$, $c = 1$

But $\hbar c$ works as a conversion factor

$$\boxed{\hbar c = 197.3 \text{ MeV} \cdot \text{fm}}$$

$1 \text{ fm} = 10^{-15} \text{ m}$

Let's go back to the hydrogen atom:

$$\left[-\nabla^2 - \frac{2\mu\alpha}{r}\right]\psi(\vec{r}) = -\gamma_B^2\,\psi(\vec{r})$$

$$\hookrightarrow \left[-\nabla^2 - \frac{2}{a_B r}\right]\psi(\vec{r}) = -\gamma_B^2\,\psi(\vec{r})$$

$$a_B = \frac{1}{\mu\alpha} \simeq \frac{1}{m_e\alpha} \simeq 268\,\text{MeV}^{-1} \simeq 5.3\cdot 10^4\,\text{fm}$$

And now notice this: a_B is the only scale here

$$\left[-\nabla^2 - \frac{2}{a_B r}\right]\psi(\vec{r}) = -\gamma_B^2\,\psi(\vec{r})$$

INPUT: a_B

OUTPUT

What does this mean?

$\gamma_B = \gamma_B(aB)$ → the wave number can only depend on aB

IT CANNOT DEPEND ON ANYTHING ELSE!

$\gamma_B \to$ momentum $a_B \to$ distance

$$\Rightarrow \boxed{\gamma_B = \frac{c_B}{a_B}} \quad \text{w/ } c_B \text{ a nomber}$$

Question: how big/small is c_B?

The same is true for everything else:

$\sqrt{\langle r^2 \rangle} \rightarrow$ mean square radius

$$\langle r^2 \rangle = \langle \psi | r^2 | \psi \rangle$$

$$= \int d^3\vec{r}\, |\psi(\vec{r})|^2 r^2$$

Then for $\sqrt{\langle r^2 \rangle}$ we should have:

$$\boxed{\sqrt{\langle r^2 \rangle} = d_B \, a_B} \quad \text{w/ } d_B \text{ a number}$$

Same question: how big/small is d_B?

With this we end the lesson:

$$\left[\gamma_B = \frac{c_B}{a_B}, \quad \sqrt{\langle r^2 \rangle} = d_B \, a_B \right]$$

So what are your guesses for c_B, d_B?

你们猜猜!